EVOLUTION OF MAGNETIC HELICITY AND ENERGY SPECTRA OF SOLAR ACTIVE REGIONS

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ABSTRACT

We adopt an isotropic representation of the Fourier-transformed two-point correlation tensor of the magnetic field for estimating magnetic energy and helicity spectra as well as current helicity spectra of individual active regions and the change of their spectral indices with the solar cycle. The departure of the spectral index of current helicity from 5/3 is analyzed, and it is found that it is lower than that of magnetic energy. There is no obvious relationship between the change of the normalized magnetic helicity and the integral scale of the magnetic field for individual active regions. The evolution of the spectral index reflects the development and distribution of various scales of magnetic structures in active regions. It is found that around solar maximum the magnetic energy and helicity spectra are steeper.

Subject headings: Sun: activity—Sun: magnetic topology—Sun: photosphere—Sun: dynamo

1. INTRODUCTION

Magnetic helicity is an important quantity that reflects the topology of the magnetic field (Woltjer 1958a,b; Taylor 1988). Pioneering studies of magnetic helicity in solar physics have been done by several authors focusing on the accumulation of magnetic helicity in the solar atmosphere (e.g. Berger & Field 1984; Chae 2001), the force-free coefficient, and the mean current helicity density in solar active regions (Seehafer 1990).

Besides the hemispheric sign distribution of large-scale helical features in active regions (Pevtsov et al. 1994; Abramenko et al. 1997), the distribution of small-scale magnetic helicity features in active regions has been analyzed by several authors (e.g. Su et al. 2009; Venkatakrishnan & Tiwari 2009; Zhang 2010). They pointed out that the existence of global twist in a sunspot — even in the absence of a net current — is consistent with a fibril structure of sunspot magnetic fields. The redistribution of magnetic helicity between different scales was argued to be the interchange of twist and writhe due to magnetic helicity conservation (cf. Zeldovich et al. 1983; Kerr & Brandenburg 1999). Furthermore, the spectral magnetic helicity distribution is important for understanding the operation of the solar dynamo (Brandenburg & Subramanian 2003). The basic characteristics of magnetic energy and helicity spectra in the active region NOAA 11158 has been analyzed by Zhang et al. (2014) under the assumption of an isotropic representation of the Fourier-transformed two-point correlation tensor of the magnetic field. The current helicity spectrum has been estimated from the magnetic helicity spectrum, again under the assumption of isotropy, and its modulus shows a $k^{-5/3}$ spectrum at intermediate wavenumbers. A similar power law is also obtained for the magnetic energy spectrum.

The variation of magnetic energy and helicity spectra of active regions with the solar cycle is also an important aspect. The observational evidence of the changes of current helicity of active regions with the solar cycles has been studied (cf. Zhang et al. 2010; Yang & Zhang 2012; Zhang & Yang 2013; Pipin & Pevtsov 2014), but changes of their spectral properties with the solar cycle remain still an open question.

In the present paper, we consider the evolution of the spectrum of magnetic helicity and its relationship with the magnetic energy from photospheric vector magnetograms of solar active regions. We also present the statistical properties of the spectrum of magnetic energy and helicity of active regions with the solar cycle.

2. BASIC FORMALISM

Let us now introduce the two-point correlation tensor, $\langle B_i(x, t)B_j(x + \xi, t) \rangle$, where $x$ is the position vector on the two-dimensional surface and angle brackets denote ensemble averaging or, in the present case, averaging over annuli of constant radii, i.e., $|\xi| = \text{const}$. Its Fourier transform with respect to $\xi$ can be written as

$$\langle \hat{B}_i(k, t)\hat{B}_j(k', t) \rangle = \Gamma_{ij}(k, t)\delta^2(k - k'),$$

where $\hat{B}_i(k, t) = \int B_i(x, t)e^{ik\cdot x}d^2x$ is the two-dimensional Fourier transform, the subscript $i$ refers to one of the three magnetic field components, and the asterisk denotes complex conjugation. Under isotropic conditions, the spectral correlation tensor $\Gamma_{ij}(k, t)$ takes the form (Zhang et al. 2014)

$$\Gamma_{ij}(k, t) = \frac{2E_M(k, t)}{4\pi k}\left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) + \frac{iM(k, t)}{4\pi k}\varepsilon_{ijk}k_k,$$

where $\hat{k}_i = k_i/k$ is a component of the unit vector of $k$, $k = |k|$ is its modulus with $k^2 = k_\perp^2 + k_\parallel^2$, while $E_M(k, t)$ and $M(k, t)$ are the magnetic energy and helicity spectra.
We emphasize that the expression for $\Gamma_{ij}$ differs from that of Moffatt [1978] by a factor $2k^2$, because we are here in two dimensions, so the differential for the integration over shells in wavenumber space changes from $4\pi k^2 \, dk$ to $2\pi k \, dk$.

To calculate the relative magnetic helicity, we define the integral scale of the magnetic field in the usual way as

$$l_M = \int k^{-1} E_M(k) \, dk \bigg/ \int E_M(k) \, dk.$$  

(4)

The realizability condition $k|H_M(k,t)| \leq 2E_M(k,t)$ (cf. Moffatt [1969]) can be rewritten in integrated form (e.g. Kahniashvili et al. 2013) as

$$\mathcal{H}_M = \int H_M \, dk \leq 2 \int k^{-1} E_M(k) \, dk = 2l_M \mathcal{E}_M. \quad (5)$$

In particular, we have $|\mathcal{H}_M(t)| \leq 2l_M \mathcal{E}_M(t)$. This allows us then to define the normalized (nondimensional) magnetic helicity,

$$r_M = \mathcal{H}_M / 2l_M \mathcal{E}_M. \quad (6)$$

which obeys $|r_M| \leq 1$. The normalized magnetic helicity varies between $-1$ and $1$ and is therefore sometimes also referred to as the relative helicity, but it must not be confused with the gauge-invariant magnetic helicity of Berger & Field [1984], which is sometimes also called relative magnetic helicity.

3. MAGNETIC HELICITY AND ENERGY SPECTRA OF INDIVIDUAL ACTIVE REGIONS

3.1. Active Region NOAA 11158

We have analyzed data from the solar active region NOAA 11158 during 12–16 February 2011, taken by the Helioseismic and Magnetic Imager (HMI) aboard the Solar Dynamics Observatory (SDO). The pixel resolution of magnetograms is about 0.5", and the field of view is $250'' \times 150''$. In our study, 600 vector magnetograms in the active region have been used.

Figure 1 shows photospheric vector magnetograms and the corresponding distribution of $h^{(z)}_C = J_z B_z$ from the vector magnetograms of that active region on different days. Here, $J_z = \partial B_y / \partial x - \partial B_x / \partial y$ is proportional to the vertical component of the current density. The superscript '(z)' on $h^{(z)}_C$ indicates that only the vertical contribution to the current helicity density is available.

We now consider magnetic energy and helicity spectra for the active region NOAA 11158. The calculated region of the field of view is $256'' \times 256''$ (i.e. $512 \times 512$ pixels). We analyze magnetic energy and helicity in the active region NOAA 11158 on 12–16 February 2011 by means of the method described above (Zhang et al. 2014).

For analyzing the basic properties of the spectra of magnetic energy and corresponding helicity in NOAA 11158, Figure 2 shows an average of spectra from the active region NOAA 11158 between 12–15 February 2011.
To study the evolution of magnetic energy and helicity spectra in solar active regions, we have also analyzed data from the solar active region NOAA 11515 during 30 June – 6 July 2012, taken by the Helioseismic and Magnetic Imager (HMI) aboard the Solar Dynamics Observatory (SDO). The pixel resolution of magnetograms is about 0.5″, and the field of view is 250″ × 150″.

### 3.2. Active Region NOAA 11515

To study the evolution of magnetic energy and helicity spectra in solar active regions, we have also analyzed data from the solar active region NOAA 11515 during 30 June – 6 July 2012, taken by the Helioseismic and Magnetic Imager (HMI) aboard the Solar Dynamics Observatory (SDO). The pixel resolution of magnetograms is about 0.5″, and the field of view is 250″ × 150″. In our study, we estimate the spectral indices $\alpha_E$ of magnetic energy and $\alpha_H$ of magnetic helicity within the wavenumber interval $1 \text{ Mm}^{-1} < k < 6 \text{ Mm}^{-1}$. Figure 4 shows the evolution of $\alpha_i$ for NOAA 11158. It is found that the minimum $\alpha_E$ is 1.1, the maximum is 2.0, and the mean value is about 1.67 as the active region developed.

Under isotropic conditions, the current helicity spectrum $H_C(k,t)$ is related to the magnetic helicity spectrum via $H_C(k,t) \sim k^2 H_M(k,t)$. In the following, we also determine the spectral indices $\alpha_C$ of current helicity, and $\alpha_{H\ell}$ of $k|H_M(k,t)|$. Figure 4 shows the evolution of $\alpha_C$ for the active region NOAA 11158. It is found that the minimum $\alpha_C$ is 0.9, the maximum is 1.7, and the mean value is about 1.6 as the scale of active region changes. This implies that the value of $\alpha_C$ of this active region is of the order of 5/3 and consistent with our previous study (Zhang et al. 2014). It also means that the mean values of $\alpha_E$ and $\alpha_C$ of this active region at the solar surface are roughly consistent with a $k^{-5/3}$ power law.

Next, we consider the spectrum of magnetic helicity and the corresponding spectral index $\alpha_H$. The minimum value of $\alpha_{H\ell}$ is 1.9, the maximum is 2.8, and the mean value is 2.65 as the active region developed.

The values of $\alpha_E$ and $\alpha_H$ in the active region reflect the characteristic distribution of different scales of magnetic helicity and energy at the solar surface, and do not really reflect the complexity of the distribution of the magnetic field with different polarities.

The evolution of $r_M$ and $l_M$ in NOAA 11158 is shown in Figure 5. The average value of $r_M$ is about 0.05, while that of $l_M$ is about 6 Mm in the developed stage of the active region. It is found that the normalized magnetic helicity $r_M$ shows a relatively complex relationship with the development of the active region and it shows a similar tendency with $H_M$ in Figure 3.

600 vector magnetograms of NOAA 11158 during 11–15 February 2011. These are comparable with that of Zhang et al. (2014), except that the fluctuations in the calculation of individual samples have been reduced by the averaging. This provides a basic estimation of the spectral distribution of magnetic energy and helicity in the active region.

Figure 3 shows the evolution of mean helicity and energy in the photosphere of the active region NOAA 11158, obtained by integrating over all $k$; see Equation (3). It is found that magnetic helicity and energy first increase and then continue to change as the active region develops. A decrease of magnetic helicity in the active region occurred on 2011 Feb. 14, while the magnetic energy did not. This suggests that the magnetic helicity in the active region does not have a monotonous relationship with the magnetic energy. This is also consistent with the trends found by Abramenko (2005) and Stenflo (2012) based on solar magnetic field observations.

![Figure 3](image1.png)  
**Fig. 3.** Evolution of photospheric magnetic helicity $H_M(t)$ (solid line) and magnetic energy $E_M(t)$ (dotted line) of active region NOAA 11158.

![Figure 4](image2.png)  
**Fig. 4.** Evolution of $\alpha_C$ (solid line) and $\alpha_E$ (dotted line) of active region NOAA 11158.

According to the theory of hydromagnetic turbulence by Goldreich & Sridhar (1995), the magnetic energy spectrum has a power law inertial range $\propto k^{-\alpha}$, where the spectral index $\alpha$ is $5/3$ (about 1.67). This was confirmed by Abramenko (2005) and Stenflo (2012) based on solar magnetic field observations.

We estimate the spectral indices $\alpha_E$ of magnetic energy and $\alpha_H$ of magnetic helicity within the wavenumber interval $1 \text{ Mm}^{-1} < k < 6 \text{ Mm}^{-1}$. Figure 4 shows the evolution of $\alpha_i$ for NOAA 11158. It is found that the minimum $\alpha_E$ is 1.1, the maximum is 2.0, and the mean value is about 1.67 as the active region developed.

Under isotropic conditions, the current helicity spectrum $H_C(k,t)$ is related to the magnetic helicity spectrum via $H_C(k,t) \sim k^2 H_M(k,t)$. In the following, we also determine the spectral indices $\alpha_C$ of current helicity, and $\alpha_{H\ell}$ of $k|H_M(k,t)|$. Figure 4 shows the evolution of $\alpha_C$ for the active region NOAA 11158. It is found that the minimum $\alpha_C$ is 0.9, the maximum is 1.7, and the mean value is about 1.6 as the scale of active region changes. This implies that the value of $\alpha_C$ of this active region is of the order of 5/3 and consistent with our previous study (Zhang et al. 2014). It also means that the mean values of $\alpha_E$ and $\alpha_C$ of this active region at the solar surface are roughly consistent with a $k^{-5/3}$ power law.

Next, we consider the spectrum of magnetic helicity and the corresponding spectral index $\alpha_H$. The minimum value of $\alpha_{H\ell}$ is 1.9, the maximum is 2.8, and the mean value is 2.65 as the active region developed.

The values of $\alpha_E$ and $\alpha_H$ in the active region reflect the characteristic distribution of different scales of magnetic helicity and energy at the solar surface, and do not really reflect the complexity of the distribution of the magnetic field with different polarities.

The evolution of $r_M$ and $l_M$ in NOAA 11158 is shown in Figure 5. The average value of $r_M$ is about 0.05, while that of $l_M$ is about 6 Mm in the developed stage of the active region. It is found that the normalized magnetic helicity $r_M$ shows a relatively complex relationship with the development of the active region and it shows a similar tendency with $H_M$ in Figure 3.

3.2. Active Region NOAA 11515

To study the evolution of magnetic energy and helicity spectra in solar active regions, we have also analyzed data from the solar active region NOAA 11515 during 30 June – 6 July 2012, taken by the Helioseismic and Magnetic Imager (HMI) aboard the Solar Dynamics Observatory (SDO). The pixel resolution of magnetograms is about 0.5″, and the field of view is 250″ × 150″. In our study,
about 840 vector magnetograms have been used.

Figure 6 shows photospheric vector magnetograms and the corresponding distribution of $h_{Cz}^{(z)} = JzBz$ from the vector magnetograms of this active region on different days. It shows the spatial distribution of magnetic field and current helicity density of this active region at the solar surface.

For analyzing the basic properties of the spectra of magnetic energy and corresponding magnetic helicity in the active region, Figure 7 shows the averaged spectra inferred from about 840 vector magnetograms of active region NOAA 11515 during 30 June – 6 July 2012. These are comparable with the result of Zhang et al. (2014) and the average spectrum of NAOO 11158 in Figure 2. Comparing the two active regions NOAA 11515 and 11158, it is found that the mean spectral configuration of magnetic energy and helicity are slightly different.

It turns out the spectral indices $\alpha_E$ and $\alpha_C$ for wavenumbers in the interval $1 < k < 6$ (Mm$^{-1}$) are similar. Figure 8 shows the evolution of $\alpha_E$ and $\alpha_C$ for this wavenumber interval in NOAA 11515. It is found that the minimum of $\alpha_C$ is 1.2, the maximum is 2.7, and the mean value is about 2.0. The minimum of $\alpha_E$ is 2.0, the maximum is 2.6, and the mean value is about 2.4. Comparing with NOAA 11158, these values are larger than those of NOAA 11158 and exceed the expected $5/3$ value.

The evolution of $r_M$ and $l_M$ in active region NOAA 11515 is shown in Figure 9. The average value of $r_M$ is about 0.22, while that of $l_M$ is about 8 Mm during the evolution of the active region. We can see that the values of the normalized magnetic helicity and the integral scale of the magnetic field decrease during the evolution of the active region – even though the area of the active region increases.

4. MAGNETIC HELICITY AND ENERGY SPECTRA OF ACTIVE REGIONS WITH SOLAR CYCLES

Long-term statistical analyses of vector magnetograms at Huairou Solar Observing Station have been achieved over recent years (Bao & Zhang 1998; Gao et al. 2008).
Zhang et al. (2010). These also provide an opportunity to analyze the evolution of the global distribution of the spectrum of magnetic fields of active regions and the relationship with the solar activity cycle, because the averaged effect of active regions is also important in the analysis of solar cycles. Figure 10 shows averaged spectra of $kH_M$ (dotted line), current helicity $H_c$ (solid line), and magnetic energy $E_M$ (dashed line) using 6629 Huairou vector magnetograms of solar active regions observed during 1988–2005. The method is similar to that for individual active regions above. Due to the relatively low spatial resolution, Figure 10 does not yield more information at high wavenumbers. It is found that the shallow slope of the spectra of magnetic energy at high wavenumbers is mainly due to observational errors of the transverse components of the magnetic field. For more information about Huairou vector magnetograms we refer to the papers by Ai & Hu (1986), Su & Zhang (2004a), Su & Zhang (2004b), and Gao et al. (2008).

We find similar magnetic helicity and energy spectra as for the individual active regions observed by HMI and the averaged one inferred from the active regions observed at Huairou Solar Observing Station. The distribution of mean current helicity of active regions inferred from Huairou vector magnetograms with sunspot butterfly diagrams has been studied by Zhang et al. (2010). It shows the same tendency as the distribution of helicity and energy spectra for the individual active regions observed by HMI and the averaged one inferred from the active regions observed at the Huairou Solar Observing Station.

To analyze the evolution of the averaged magnetic helicity and energy spectra of solar active regions, we show in Figure 11 the latitudinal and temporal dependence of $\alpha_C$ and $\alpha_E$ with the solar cycle in the spectral range $0.2 \text{ Mm}^{-1} < k < 0.6 \text{ Mm}^{-1}$. The slopes of the spectra do not change systematically with latitude as one averages the spectra of active regions for 1988–2005. The spectral indices are $\alpha_{kH} \approx 2.2$, $\alpha_C \approx 1.1$, and $\alpha_E \approx 1.4$.

Figure 11 shows the temporal variation of the slopes of the spectra of magnetic energy and helicity of active regions between 1988 and 2000. These slopes show significant correlation with sunspot number. High values occur during 1990–1992 and 2000–2003, and low values during 1995. These are consistent with the periods of solar maximum and minimum, respectively. The correlation coefficient between the slopes of current helicity and sunspot numbers is 0.730 and that between magnetic energy and sunspot number is 0.827. Note also that the magnetic energy during solar solar maximum is high. Furthermore, the correlation coefficient between the slopes of current helicity and sunspot number changes to 0.831 if one takes the sunspot number delayed by one year. It is
consistent with the observational result by Zhang et al. (2010), that the maximum of mean current helicity of active regions tends to be delayed compared with that of sunspot number. A similar piece of evidence is that the complex magnetic configuration of active regions tends to occur in the decaying phase of solar cycle 23 (after 2002); see Guo et al. (2010).

Figure 12 shows the temporal evolution of the integral scale $l_M$ of the magnetic field of solar active regions inferred by 6629 Huairou vector magnetograms during 1988–2005. The correlation coefficient between the integral scale of the magnetic field, inferred from Equation (4), and sunspot numbers is 0.802. The average value of the integral scale of magnetic energy is about 8 Mm during solar maximum and 6 Mm during solar minimum for our calculated active regions. These dependencies are consistent with the finding that large-scale magnetic patterns of active regions tend to occur near solar maximum.

Figure 12 shows that the averaged photospheric normalized magnetic helicity $r_M$ of active regions obtained from Equation (6) correlates with the solar cycle (as measured by the sunspot number), except after 2003. The high departure of the mean relative magnetic helicity during 2003–2005 is roughly consistent with the high complexity of magnetic fields of active regions obtained by Guo et al. (2010), based on the analyses of MDI longitudinal magnetograms, even if different data sets are used.

5. DISCUSSIONS

In this study, the HMI and Huairou vector magnetograms have been used to estimate the spectra of magnetic energy and helicity of solar active regions. In addition, temporal changes of the magnetic energy spectra of active regions and an evolution with the solar cycle have also been found. There is an uncertainty regarding the relationship between the observational resolution of the magnetic field and the spectral shape at large wavenumbers, because we use vector magnetograms of different spatial resolutions to analyze the evolution of the spectral distributions of magnetic energy at the different times. The lower spatial resolution of vector magnetograms of ground-based observations implies a source of error in the spectrum of the magnetic field at high wavenumbers.

To estimate the possible errors in the calculation of the magnetic spectrum due to the low spatial resolution of observational magnetic fields by Huairou vector magnetograms, Figure 13 shows mean spectra of magnetic energy, as well as magnetic and current helicity, and the evolution of the spectral indices $\alpha_C$ and $\alpha_E$ for wavenumbers in the active region NOAA 11158, whose pixel size of the analyzed region of the HMI vector magnetograms have been downsampled from $512 \times 512$ to $128 \times 128$. The pixel resolution is $2'' \times 2''$ and it is then almost the same as that of the Huairou vector magnetograms. The same tendency is found for the magnetic energy spectra as in Figure 2. The high noise in the time series of $\alpha_C$ and $\alpha_E$ in Figure 4 is now reduced. The mean value of $\alpha_E$ is about 1.82 and that of $\alpha_C$ is about 1.34 for $0.4 \text{Mm}^{-1} < k < 2.0 \text{Mm}^{-1}$, while the values obtained for the original resolution in Figure 4 are 1.62 and 1.51, respectively. This implies that the resolution of the observational vector magnetograms is still problematic in the diagnostics of the spectra of the magnetic field in the detail study. This may affect the analyzes of the changes of the spectral slopes with the solar cycle when using the Huairou vector magnetograms.

6. CONCLUSIONS

We have applied the technique of Zhang et al. (2014) to estimate magnetic energy and helicity spectra using vector magnetogram data at the solar surface. We have made use of the assumption that the spectral two-point
correlation tensor of the magnetic field can be approximated by its isotropic representation. In this paper, we have analyzed the evolution of magnetic energy and helicity spectra in active regions and have also analyzed the changes with the solar cycle. Our major results are the following:

1) The values of $\alpha_E$ and $\alpha_C$ of solar active regions are of the order of $5/3$, although $\alpha_C$ is slightly lower than $\alpha_E$, i.e., the current helicity spectrum is slightly shallower than the magnetic energy spectrum. The values are still roughly compatible with a Kolmogorov power law (Kolmogorov 1941; Obukhov 1941) and the Iroshnikov–Kraichnan spectrum (Iroshnikov 1963; Kraichnan 1965). We have also found a systematic change of $\alpha_E$ and $\alpha_C$ with the development of active regions, which reflects their structural changes.

2) There is no the obvious relationship between the change of the photospheric normalized magnetic helicity ($r_M$) and the integral scale of the magnetic field ($l_M$) of individual active regions. This means that the increase of the mean scale of magnetic structures does not implies that the magnetic helicity in the active regions increases.

3) We have found that there is a correlation between the variation of the spectra of magnetic energy and helicity of solar active regions with solar cycles. This reflects that the characteristic scales and the intensity of the magnetic fields of active regions changes with the solar cycle.

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