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Cross helicity and turbulent magnetic diffusivity in the solar convection zone

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Abstract In a density-stratified turbulent medium the cross helicity $\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle$ is considered as a result of the interaction of the velocity fluctuations and a large-scale magnetic field. By means of a quasilinear theory and by numerical simulations we find the cross helicity and the mean vertical magnetic field anti-correlated. In the high-conductivity limit the ratio of the helicity and the mean magnetic field equals the ratio of the magnetic eddy diffusivity and the (known) density scale height. The result can be used to predict that the cross helicity at the solar surface exceeds the value of 1 Gauss · km/s. Its sign is anti-correlated with that of the radial mean magnetic field. Alternatively, we can use our result to determine the value of the turbulent magnetic diffusivity from observations of the cross helicity.

Keywords: Sun: magnetic field – Magnetohydrodynamics (MHD)

1. Introduction

Dynamo theory for convective zones needs to know both the values of the α-effect and the eddy diffusivity. The α-effect is strongly related to the kinetic helicity which has different signs at different hemispheres. Almost all of the theoretical calculations for rotating stratified turbulence lead to negative helicity (i.e. positive α-effect) for the northern hemisphere and positive helicity (i.e. negative α-effect) for the southern hemisphere. Despite all of the complications to measure the helicity on the solar surface, a new result has recently been presented by Komm, Hill and Howe (2008). They do indeed find negative (positive) values

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for the kinetic helicity in the northern (southern) hemisphere. This result is based on a ring-diagram analysis of GONG data. The values remained constant as long as the flux did not exceed 10 Gauss. Using the observation of the DIV-CURL correlation, $\mathcal{C} = \langle (u_{,x} + v_{,y})(v_{,x} - u_{,y}) \rangle$, which is proportional to the kinetic helicity (Rüdiger, Brandenburg, and Pipin, 1999; see Rüdiger and Hollerbach, 2004 for more details), Duvall and Gizon (2000) found $\mathcal{C}$ to be negative (positive) in the northern (southern) hemisphere, as derived from the horizontal velocity components of mesogranulation patterns. Egorov, Rüdiger, and Ziegler (2004) simulated these observations with the NIRVANA code and reproduced them with Tayler numbers as small as 10^3.

By use of the finding of Keinigs (1983) the current helicity and the α-effect are anti-correlated, so one also can derive the sign of the α-effect by observation of the current helicity $\langle \boldsymbol{J}' \cdot \boldsymbol{B}' \rangle$. Seehafer (1990) started to observe the current helicity at the solar surface showing that it is negative (positive) in the northern (southern) hemisphere. Again the α-effect is found to be positive (negative) in the northern (southern) hemisphere.

The numerical value of the helicity derived by Komm *et al.* (2008) is of the order of 10^{-7} cm/s^2. This turns out to be very small, because that the resulting α-effect is below 1 cm/s. By comparison, Käpylä, Korpi and Brandenburg (2009) find $\alpha \simeq 0.03 u_{\rm rms}$ near the surface from their convection simulations. With $u_{\rm rms} \simeq 300$ m/s this corresponds to 10 m/s. The maximal α-value in their box center is of the order of 0.3 $u_{\rm rms}$. This highlights a major discrepancy between theory and observations or, at least, a difficulty in determining α from observations.

The empirical definition of the turbulent magnetic diffusion seems to be more straightforward. The decay of non-permanent magnetic structures such as sunspots or larger active regions lead to numerical values of the turbulent magnetic diffusivity. One finds $\eta_{\rm T} \simeq 10^{11}$ cm^2/s from sunspot decay (Stix 1989) or $\eta_{\rm T} \simeq 10^{12}$ cm^2/s from the decay of active regions (Schrijver and Martin, 1990). These values are smaller than the value of $3 \cdot 10^{12}$ cm^2/s, which results from the widely used formula $\eta_{\rm T} \sim 0.3 u_{\rm rms} \ell_{\rm corr}$ with correlation or mixing length $\ell_{\rm corr}$ and parameter values taken close to the surface. There is no possibility until now to observe the turbulent diffusivity on the solar surface for the quiet Sun where the magnetic quenching of this quantity by large-scale magnetic fields is negligible. We shall demonstrate in the present paper that there is a rather simple possibility to observe the magnetic diffusivity even in the presence of very weak magnetic fields (< 10 Gauss), for which quenching should be negligible.

2. Mean-field electrodynamics

Let $\boldsymbol{u}' = \boldsymbol{u} - \langle \boldsymbol{u} \rangle$ and $\boldsymbol{B}' = \boldsymbol{B} - \langle \boldsymbol{B} \rangle$ be the fluctuations of velocity and magnetic field about an average value denoted by angular brackets. The mean-field dynamo theory of cosmic magnetic fields is based on the relation

$$\langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle = \alpha \langle \boldsymbol{B} \rangle - \beta \langle \boldsymbol{J} \rangle \tag{1}$$

between the turbulent electromotive force $\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle$ and the mean-field quantities $\langle \boldsymbol{B} \rangle$ and $\langle \boldsymbol{J} \rangle$, where $\boldsymbol{J}$ is the mean current density. Note the basic difference between the quantities α and β in that β is a scalar while α is a pseudoscalar. For rotating stars a pseudoscalar can be formed by use of the basic rotation rate, e.g., $\alpha \propto (\boldsymbol{g} \cdot \boldsymbol{\Omega})$ with $\boldsymbol{g}$ as the radial direction as the only remaining preferred direction apart from $\boldsymbol{\Omega}$. Hence, the amplitude of the α-effect must mainly be influenced by the Coriolis number

$$\Omega^* = 2\tau_{\rm corr}\Omega, \tag{2}$$

where $\tau_{\rm corr}$ is the correlation time of the dominating mode of turbulence. However, Ω^* is very small at the solar surface so that the α-effect in Equation (1) cannot be observed directly.

The β in Equation (1) exists even in nonrotating plasmas. It is thus not governed by the Coriolis number Ω^* and is therefore not small by comparison. It is, however, not possible to observe by direct means the mean current density $\langle \boldsymbol{J} \rangle$ at the solar surface. The decay of sunspots should provide good estimates for β when the induction equation is solved by using Equation (1); and the time-dependent solutions are compared with the observations (Krause and Rüdiger, 1975). Though successful, this procedure cannot serve as a proof of the existence of Equation (1). We must conclude, therefore, that the basic Equation (1) cannot be tested with observations taken from the solar surface. This is an unsatisfying situation in that Equation (1) is a fundamental relation of a whole branch of cosmic MHD and, of course, there is no better laboratory than the Sun to probe such basic relations.

Fortunately, the situation is quite different for another correlation between fluctuations of flow and field, namely the cross helicity $\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle$, which itself is a pseudoscalar. It is straightforward to formulate the relation

$$\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle = \alpha_c \langle \boldsymbol{g} \cdot \boldsymbol{B} \rangle - \beta_c \langle \boldsymbol{\Omega} \cdot \boldsymbol{J} \rangle \tag{3}$$

similar to Equation (1). The α_c-effect does *not* run with the Coriolis number Ω^*. Similar to the α-effect in (1) the α_c in (3) is of the dimension of a velocity but this velocity should be much faster than the corresponding α in Equation (1). As the second term on the RHS of (3) only exists in the presence of rotation, it will be negligibly small at the solar surface.

In summary, by simple reasons the observations of the cross correlation $\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle$ at the solar surface should give a realistic chance to confirm the existence of relations that are typical for mean-field electrodynamics.

3. Quasilinear theory of cross helicity

In this section we derive the symmetric part $\langle u_i' B_j' \rangle^{\rm s} = (\langle u_i' B_j' \rangle + \langle u_j' B_i' \rangle)/2$ of the cross correlation tensor $\langle u_i' B_j' \rangle$. The pseudotensor $\langle u_i' B_j' \rangle$ can be finite only in presence of a mean magnetic field $\boldsymbol{B}$ and for inhomogeneous fluids. The required inhomogeneity can be due to stratification of density or turbulent intensity as well as the inhomogeneity of the mean field itself.

The turbulent flow is assumed anelastic, so that $\mathrm{div}(\rho\boldsymbol{u}') = 0$. It is convenient to use the Fourier transformation of the momentum density $\boldsymbol{m} = \rho\boldsymbol{u}'$, i.e.

$$\boldsymbol{m}(\boldsymbol{r}, t) = \int \hat{\boldsymbol{m}}(\boldsymbol{k}, \omega)\ \mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\mathrm{d}\boldsymbol{k}\ \mathrm{d}\omega, \tag{4}$$

and similarly for the fluctuation of the magnetic field. The linearized equation for magnetic fluctuations in terms of the Fourier amplitudes reads

$$\begin{aligned}(-\mathrm{i}\omega\ +\ \eta k^2)\hat{B}'_i(\boldsymbol{k},\omega) &= \\ =\ \mathrm{i}k_j \int \Bigg(&\hat{m}_i(\boldsymbol{k}-\boldsymbol{k}', \omega-\omega')\left(\frac{\hat{B}_j}{\rho}\right)(\boldsymbol{k}', \omega') - \\ -\ &\hat{m}_j(\boldsymbol{k}-\boldsymbol{k}', \omega-\omega')\left(\frac{\hat{B}_i}{\rho}\right)(\boldsymbol{k}', \omega')\Bigg)\ \mathrm{d}\boldsymbol{k}'\mathrm{d}\omega',\end{aligned} \tag{5}$$

where $\hat{\boldsymbol{B}}$ is the Fourier transform of the mean magnetic field.

The spectral tensor of the momentum density that accounts for the stratification of the turbulence to the first order terms reads

$$\begin{aligned}\langle\hat{m}_i(\boldsymbol{z},\omega)\hat{m}_j(\boldsymbol{z}',\omega')\rangle &= \delta(\omega+\omega')\frac{\hat{q}(k,\omega,\boldsymbol{\kappa})}{16\pi k^2} \\ &\times\left(\delta_{ij} - k_i k_j/k^2 + (\kappa_i k_j - \kappa_j k_i)/(2k^2)\right),\end{aligned} \tag{6}$$

where $\boldsymbol{k} = (\boldsymbol{z}-\boldsymbol{z}')/2,\ \boldsymbol{\kappa} = \boldsymbol{z}+\boldsymbol{z}'$, $\hat{q}$ is the Fourier transform of the local spectrum,

$$q(k,\omega,\boldsymbol{r}) = \rho^2 E(k,\omega,\boldsymbol{r}) = \int \hat{q}(k,\omega,\boldsymbol{\kappa})\mathrm{e}^{\mathrm{i}\boldsymbol{\kappa}\cdot\boldsymbol{r}}\ \mathrm{d}\boldsymbol{\kappa}, \tag{7}$$

so that

$$\langle u'^2\rangle = \int\limits_0^\infty\int\limits_0^\infty E(k,\omega,\boldsymbol{r})\ \mathrm{d}k\mathrm{d}\omega. \tag{8}$$

Derivation of the cross correlation yields

$$\begin{aligned}\langle u'_i B'_j\rangle^{\mathrm{s}} &= \frac{1}{2}\eta_{\mathrm{T}}\left(G_i\langle B_j\rangle + G_j\langle B_i\rangle\right) + \left(\frac{1}{10}\eta_{\mathrm{T}} + \frac{4}{15}\hat{\eta}\right)\delta_{ij}\left(\boldsymbol{U}\cdot\langle\boldsymbol{B}\rangle\right) + \\ &+ \left(\frac{1}{10}\eta_{\mathrm{T}} - \frac{1}{15}\hat{\eta}\right)\left(U_i\langle B_j\rangle + U_j\langle B_i\rangle\right) - \left(\frac{3}{10}\eta_{\mathrm{T}} + \frac{2}{15}\hat{\eta}\right)\left(\langle B_{j,i}\rangle + \langle B_{i,j}\rangle\right),\end{aligned} \tag{9}$$

where $\boldsymbol{G} = \boldsymbol{\nabla}\log\rho$ and $\boldsymbol{U} = \boldsymbol{\nabla}\log\langle u'^2\rangle$ are the gradients of density and turbulent intensity and

$$\eta_{\mathrm{T}} = \frac{1}{3}\int\limits_0^\infty\int\limits_0^\infty \frac{\eta k^2 E}{\omega^2 + \eta^2 k^4}\mathrm{d}k\ \mathrm{d}\omega, \tag{10}$$

$$\hat{\eta} = \int\limits_0^\infty\int\limits_0^\infty \frac{\eta k^2\omega^2 E}{(\omega^2 + \eta^2 k^4)^2}\mathrm{d}k\ \mathrm{d}\omega. \tag{11}$$

Here $\eta = 1/\mu_0\sigma$ is the molecular magnetic diffusivity. Both quantities run with Rm for low conductivity ($\sigma \to 0$) and become finite for high conductivity ($\eta \to 0$). From the cross correlation tensor (9) the cross helicity

$$\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle = \eta_{\rm T} \left(\boldsymbol{G} \cdot \langle \boldsymbol{B} \rangle \right) + \left(\frac{\eta_{\rm T}}{2} + \frac{2\hat{\eta}}{3} \right) \left(\boldsymbol{U} \cdot \langle \boldsymbol{B} \rangle \right) \tag{12}$$

is obtained. Note that for uniform density ($G = 0$) the cross helicity (12) can be written as a divergence $\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle = \mathrm{div}\, [(\eta_{\rm T}/2 + 2\hat{\eta}/3) \langle \boldsymbol{B} \rangle]$. In that case it integrates to zero over the volume of a turbulent body if the normal component of $\langle \boldsymbol{B} \rangle$ on the surface is zero. The same condition is required for the global conservation of cross helicity. Density stratification, however, violates cross helicity conservation.

Current observations only supply the correlation $\langle u'_r B'_r \rangle$. From Equation (9) we find

$$\langle u'_r B'_r \rangle = \eta_{\rm T} G \langle B_r \rangle - \left(\frac{3\eta_{\rm T}}{10} + \frac{2\hat{\eta}}{15} \right) \left(2 \frac{\partial \langle B_r \rangle}{\partial r} - U \langle B_r \rangle \right), \tag{13}$$

where $G = G_r$ and $U = U_r$ are the only non-zero radial components of the stratification vectors. Further simplifications can be obtained by using the mixing-length approximation for the turbulence spectrum,

$$E(k, \omega, \boldsymbol{r}) = 2 \langle u'^2 \rangle \delta \left(k - \ell^{-1} \right) \delta(\omega), \quad \eta = \ell^2 / \tau_{\rm corr}, \tag{14}$$

where ℓ is mixing length and $\tau_{\rm corr}$ is the correlation time. It yields

$$\langle u'_r B'_r \rangle = \eta_{\rm T} \left(G \langle B_r \rangle - \frac{3}{5} \frac{\partial \langle B_r \rangle}{\partial r} + \frac{3}{10} U \langle B_r \rangle \right). \tag{15}$$

The result can be explained as follows. A rising fluid element $u'_r > 0$ expands so that B'_r has the opposite sign as $\langle B_r \rangle$. The fluid particles which go down, $u'_r < 0$, compress and B'_r has the same sign as B_r. The sign of the product $u'_r B'_r$ is opposite to $\langle B_r \rangle$ in both cases – in accord with the first term on the right hand side (RHS) of Equation (15); note the negativity of G. An upward divergence of the mean field reduces the effect of density stratification. This is realized by the second term on the RHS of Equation (15). The third term shows that also the non-uniformity of the turbulent intensity makes a contribution. However, the contribution of density stratification is probably dominant. Therefore, a finite cross correlation (15) indicates the presence of a large-scale radial field of the opposite sign.

The leading term on the RHS of Equation (15) is due to the density gradient. The resulting relation then reads

$$\frac{\langle u'_r B'_r \rangle}{\langle B_r \rangle} = -\frac{\eta_{\rm T}}{H_\rho}. \tag{16}$$

The magnetic eddy diffusivity can thus be determined if the LHS of (16) is observed and the density scale height H_ρ is known from numerical models of the solar atmosphere.

4. Numerical simulation

It is straightforward to verify the validity of Equation (16) using numerical simulations of isothermally stratified forced turbulence in a layer with constant gravity, $\boldsymbol{g} = (0, 0, -g)$ in Cartesian coordinates. In that case the scale height, $H_\rho = c_\mathrm{s}^2/g$, is constant.

We perform simulations in a cubic domain of size L^3, so the minimal wavenumber is $k \equiv k_1 = 2\pi/L$. We solve the governing equations of compressible magnetohydrodynamics with an isothermal equation of state. The flow is driven by a random forcing function consisting of non-helical waves with wavenumbers whose modulus lies in a narrow band around an average wavenumber k_f. We arrange the amplitude of the forcing function such that the RMS Mach number is around 0.1 or less, so the effects of compressibility are negligible.

In all our runs we adopt stress-free pseudo-vacuum boundary conditions on the top and bottom boundaries, i.e. the horizontal magnetic field vanishes. The magnetic field is expressed in terms of the vector potential $\boldsymbol{A}$ as $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{\nabla} \times \boldsymbol{A}$, where $\boldsymbol{B}_0 = (0, 0, B_{0z} =$ const is the imposed vertical field which is fixed for each run. The simulations were performed with the Pencil Code[1], which uses sixth-order explicit finite differences in space and third-order accurate time stepping method (Brandenburg and Dobler, 2002). A numerical resolution of up to 256^3 meshpoints was used, depending on the value of the magnetic Reynolds number.

We perform simulations for a number of different parameter combinations. The parameters that are being varied include the strength of the imposed vertical field B_z, the forcing wavenumber k_f, the gravitational acceleration g, and hence H_ρ, and the values of the magnetic diffusivity. We express these quantities in non-dimensional form and define the magnetic Reynolds number as

$$\mathrm{Rm} = \frac{u_\mathrm{rms}}{\eta k_\mathrm{f}}. \tag{17}$$

The strength of the magnetic field is characterized by the mean equipartition field strength,

$$B_\mathrm{eq} = \sqrt{\mu_0 \langle \rho \rangle}\, u_\mathrm{rms}, \tag{18}$$

which is of order 1000 Gauss at the solar surface. We determine the cross helicity, $\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle$, as a volume average. In order to relate this to Equation (16) we also need to estimate the value of the turbulent magnetic diffusivity. Earlier work by Sur, Brandenburg, and Subramanian (2008) showed that, to a good approximation, η_T can be estimated by

$$\eta_\mathrm{T} \approx \eta_\mathrm{T0} \equiv u_\mathrm{rms}/3k_\mathrm{f}, \tag{19}$$

provided $\mathrm{Rm} \gg 1$, i.e. in the high-conductivity approximation. In a number of cases we have verified the validity of this approximation also for the stratified runs shown here.

[1] `http://pencil-code.googlecode.com`

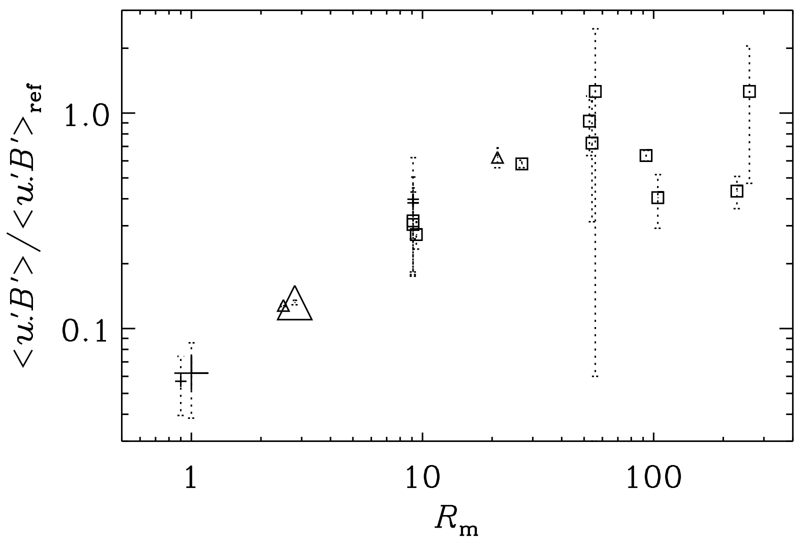

Figure 1. Dependence of the normalized cross helicity on Rm for various field strength $B_z/B_{\mathrm{eq}} < 0.1$, $\mathrm{Pm} = 1$, $k_{\mathrm{f}}/k_1 = 2.2$, and $H_\rho k_1 = 2.5$.

We present the value of $\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle$ in non-dimensional form by dividing by a reference value defined after (16) as

$$\langle \boldsymbol{u}' \cdot \boldsymbol{B}' \rangle_{\mathrm{ref}} = -\frac{\eta_{\mathrm{T}0} B_0}{H_\rho}. \tag{20}$$

For small Rm the normalized cross helicity depends on Rm (see Figure 1) but it reaches unity for large Rm. It is the expected behavior as (19) gives only a good approximation for (10) for the case of high conductivity, i.e. for $\sigma \to \infty$. In the opposite case of $\sigma \to 0$ the expression (10) vanishes so that the small numbers of the lower-left corner of Figure 1 become understandable. For small values of Rm we have $\eta_{\mathrm{T}} \propto \mathrm{Rm}$. For the largest values of Rm the error bars for the numerical results are larger. This is mainly because those simulations require larger numerical resolution and long run times become prohibitive.

Figure 2 shows the dependence of the normalized cross helicity, defined by the ratio $\langle \boldsymbol{u} \cdot \boldsymbol{B} \rangle / \langle \boldsymbol{u} \cdot \boldsymbol{B} \rangle_{\mathrm{ref}}$, on the normalized field strength B_{0z}/B_{eq}. Note that the cross helicity is quenched by nearly a factor of 10 for $B_{0z} \approx B_{\mathrm{eq}}$.

5. Conclusions

We have shown that nonrotating turbulence at the top of the solar convection zone under the influence of a vertical magnetic field yields a finite cross helicity. The only condition is the existence of a density stratification which enters

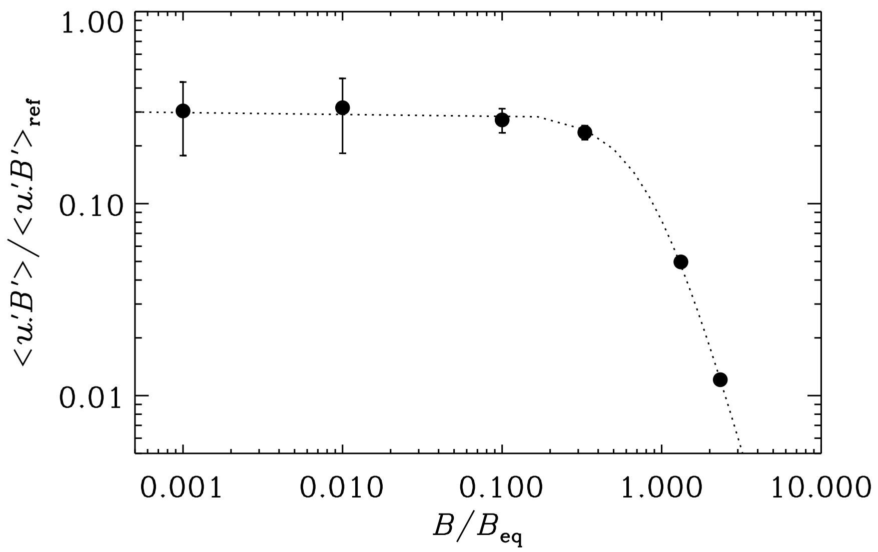

Figure 2. Dependence of the normalized cross helicity on the normalized vertical field strength for $\mathrm{Rm} = 10$, $\mathrm{Pm} = 1$, $k_\mathrm{f}/k_1 = 2.2$, and $H_\rho k_1 = 2.5$. The dotted line corresponds to the graph of $0.3/[1 + (\langle \boldsymbol{B} \rangle / B_\mathrm{ref})^2]^{3/2}$ with $B_\mathrm{ref} = 0.85\, B_\mathrm{eq}$.

the induction equation via the anelastic relation $\mathrm{div}(\rho \boldsymbol{u}') = 0$. The Boussinesq approximation cannot be used. The effect only exists in the high-conductivity limit, i.e. for sufficiently large magnetic Reynolds numbers (see Figure 1). The radial magnetic field, on the other hand, must be weak enough to remain passive so that it does not dominate the flow. Figure 2 shows that the maximum field is given by B_eq which is *much* higher than the mean vertical field, which is of the order of a few Gauss on the Sun.

To estimate the value of the cross helicity at the solar surface we shall assume a density scale height of 100 km. Then one finds from (16) that

$$\langle u'_r B'_r \rangle \simeq -\frac{\langle B_r \rangle}{1\mathrm{Gauss}} \frac{\eta_{12}}{H_7} \text{ Gauss km/s.} \tag{21}$$

The magnetic diffusivity has been used in the form $\eta_\mathrm{T} = 10^{12} \cdot \eta_{12}$ and the density scale height as $H_\rho = 100 \cdot H_7$ km. We thus predict the existence of a cross helicity of more than 1 Gauss km/s which is anti-correlated to the mean radial magnetic field, i.e.

$$\langle u'_r B'_r \rangle \ \langle B_r \rangle < 0. \tag{22}$$

For a dipolar background field the sign of the cross helicity will differ for both hemispheres.

The relation (16) can also be used to measure the magnetic diffusivity if the cross helicity is known by observations. In order to find the cross helicity one has only to correlate observed flow fluctuations with observed magnetic fluctuations. Together with the calculated mean value of the radial magnetic field Equation (16) provides the unknown quantity $\eta_{\rm T}$. We hope that such an analysis of the observations using, for example, data from the Japanese Hinode satellite will soon provide supporting evidence for an anti-correlation between $\langle u_r' B_r' \rangle$ and $\langle B_r \rangle$, and that meaningful value of $\eta_{\rm T}$ can be obtained in that way.

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