Quantum phases of helium

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OUTLINE

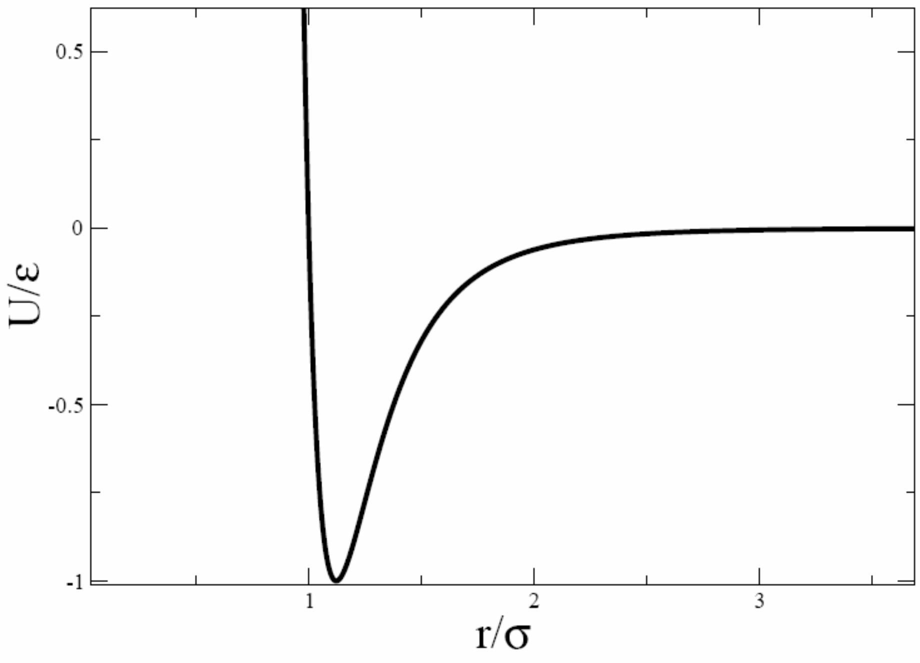
- Superfluids and supersolids
- The Chan-Kim experiment
- New phase diagram of ^4He
- Interpretations
- He in porous media
- Models, analogies, scaling

H and He

- Lightest atoms, H, He, most quantum
- ^{4}He has full electronic outer shell
- hence weak potential (compared to H)
- Lennard-Jones potential between atoms

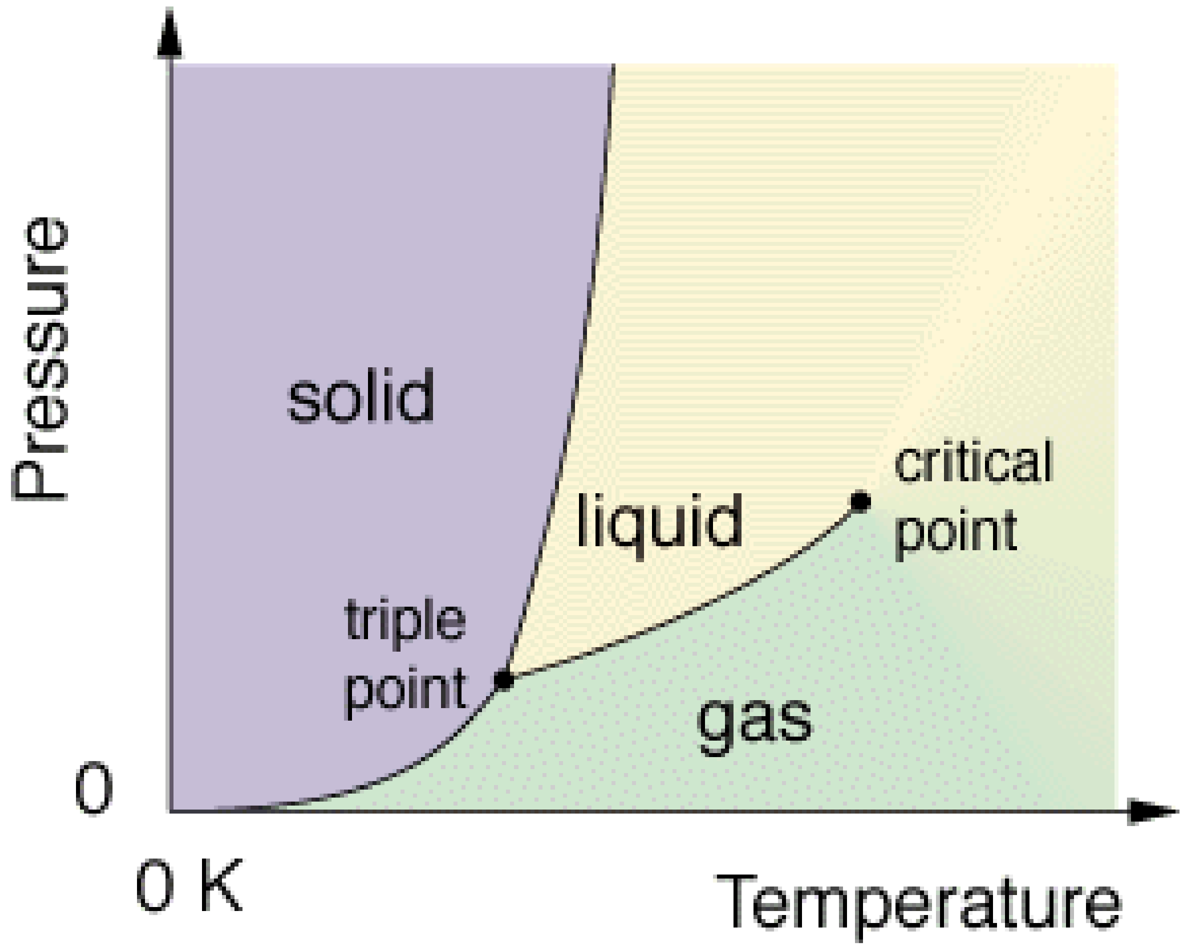
$$U(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

$$\varepsilon = 10.22K, \sigma = 2.556Å$$



Typical phase diagram

All substances (except He) solidify at T=0K



Phase diagram of ^{4}He

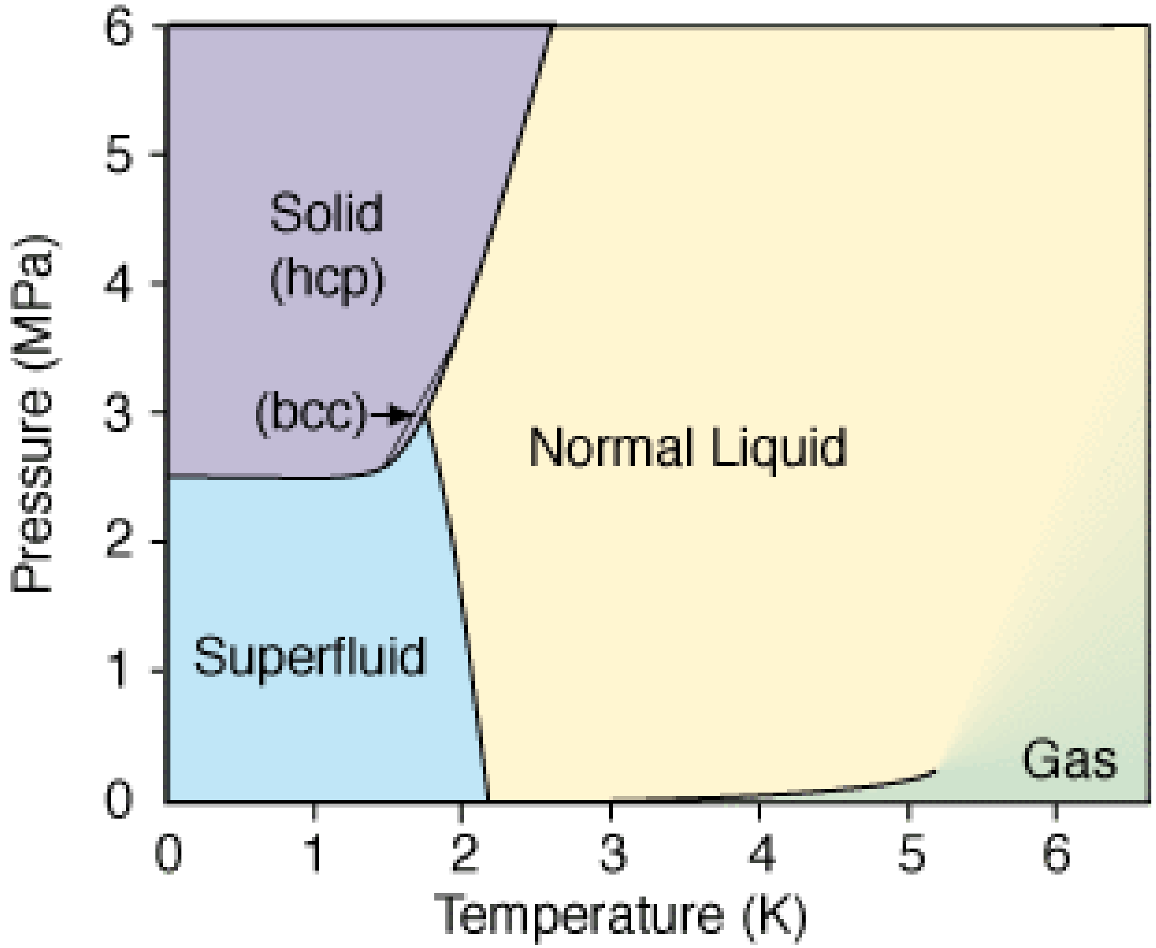
•Superfluidity as a BEC was first suggested by Fritz London.

•Condensate fraction at T=0 is 10%

•Superfluid fraction

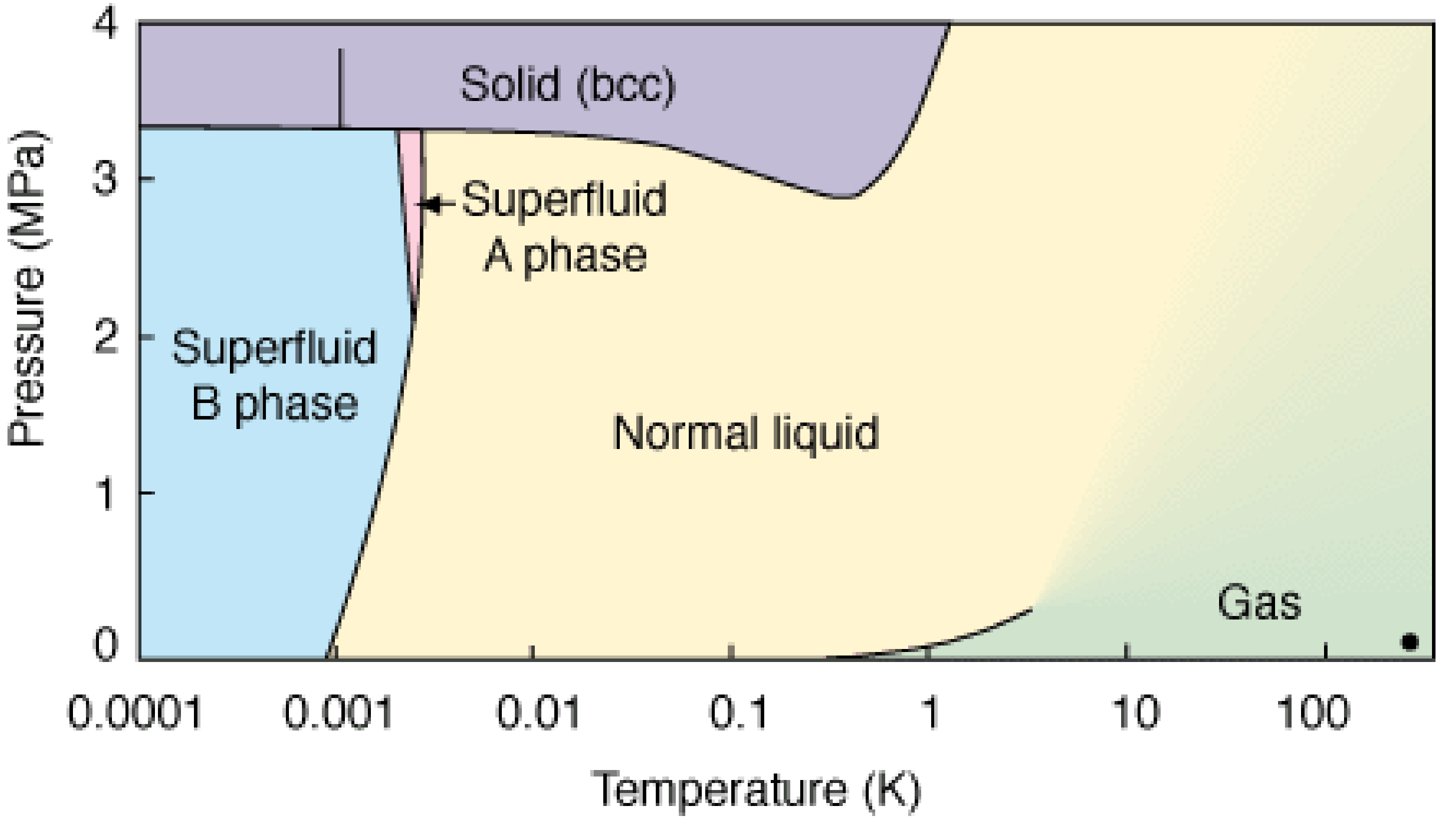
$$n_s = \rho_s / \rho$$

at T=0 is 100%

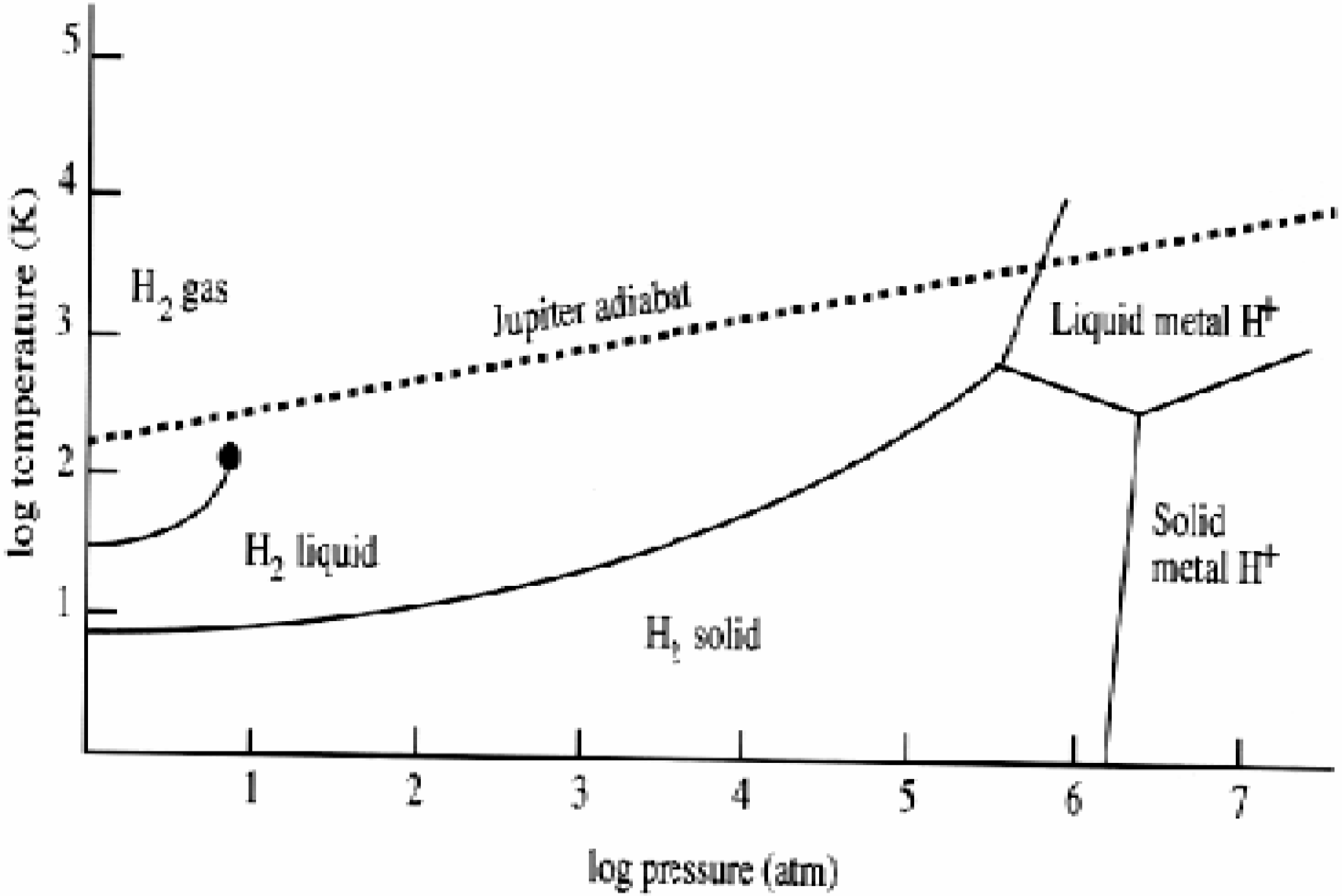


Phase diagram of ^3He

- ^3He is a fermion and has a quite different phase diagram



Hydrogen phase diagram



Anomalous melting of ^{4}He solid

- Classical Lindemann criterion for ^{4}He x-ray-diffraction measurements

$$\gamma_{4\mathrm{He}} = \frac{\sqrt{\langle u^2 \rangle}}{a} = 0.28$$

$$\gamma_{\mathrm{melting}} = 0.1 - 0.15$$

L. K. Moleko, H. R. Glyde, PRL 54, 901 (1985)
C. A. Burns, E. D. Isaacs, PRB 55, 5767 (1997)

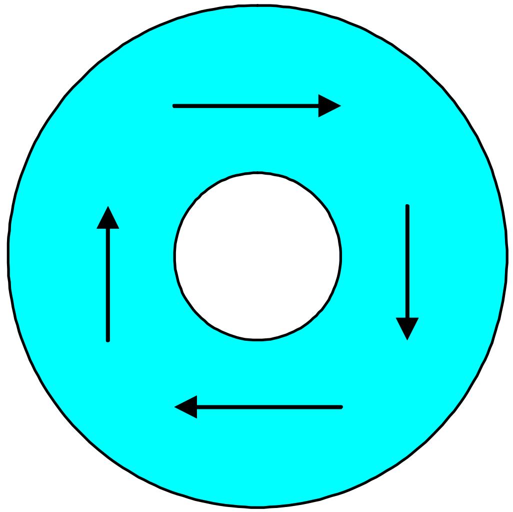
Superfluidity in ^{4}He

- Macroscopic wavefunction

$$\Psi = |\Psi| e^{i\theta} \quad \vec{j} = \rho_s \vec{v}_s \quad \vec{v}_s = \frac{\hbar}{m} \nabla \theta$$

Persistent current in a ring demonstrates zero dissipation

Can flow through pores and cracks



Superfluid and supersolid

- Solid: broken translation symmetry

$$n(\mathbf{r}) = \left\langle \psi^{+}(\mathbf{r})\psi(\mathbf{r}) \right\rangle = n(\mathbf{r}+\mathbf{R})$$

- Superfluid: ODLRO, atoms are in phase

$$\left\langle \psi^{+}(\mathbf{r})\psi(\mathbf{r}') \right\rangle \rightarrow \Psi(\mathbf{r})^{*}\Psi(\mathbf{r}')$$

$$\Psi(\mathbf{r}) = \left|\Psi(\mathbf{r})\right| e^{i\theta(\mathbf{r})}$$

- Supersolid: solid with ODLRO
  Or: superfluid with density wave

Phase-particle number duality

- Phase and particle number N are conjugate variables: $\Delta N \Delta \theta \geq \hbar$
- Localized solid: N certain, phase completely uncertain, no ODLRO
- Superfluid with localized (definite) phase: N completely uncertain, not solid
- Supersolid: both N and phase order, but with nonzero uncertainties

Non Classical Rotational Inertia

- Superfluid liquid helium fails to rotate with the container: it has NCRI (I=angular momentum/angular frequency)

$$I(T) = I_{\text{classical}}\left(1 - \frac{\rho_s(T)}{\rho}\right) \quad \text{or} \quad \frac{\rho_s(T)}{\rho} = \frac{I_{\text{classical}} - I(T)}{I_{\text{classical}}}$$

- Fritz London, Superfluids (Wiley, New York, 1954), vol. II, p. 144.
- Verified experimentally by G. B. Hess, W. M. Fairbank, PRL **19**, 216 (1967).

Reatto-Chester proposal

- In 1967, Reatto and Chester proposed that solid ^{4}He might exhibit superfluidity.

L. Reatto and G. V. Chester, Phonons and the properties of a Bose system, *Phys. Rev.* **155**, 88 (1967).

Andreev-Lifshitz suggestion: supersolid with defect condensate

- Large zero point motion of He atoms leads to defects (vacancies) in solid ground state
- Andreev-Lifshitz proposed BEC of mobile defects in the He solid

A. F. Andreev, I. M. Lifshitz, Sov. Phys. JETP **29,** 1107 (1969)

Existence of a supersolid

- Chester: If solid ^{4}He can be described by a Jastrow-type wavefunction, which is often used to describe liquid helium, then crystalline order with a finite vacancy concentration and BEC can coexist.
- **Without vacancies and/or interstitials, solids are insulators.**

G.V. Chester, PRA 2, 256 (1970)

Prokof'ev and Svistunov, PRL 94, 155302 (2005)

Leggett suggestion

- Ideal method to detect superflow: let solid helium to undergo dc or ac rotation to look for evidence of NCRI
  Leggett, PRL **25**, 1543 (1970)
- By a variational method Leggett estimated the upper limit of the supersolid fraction to

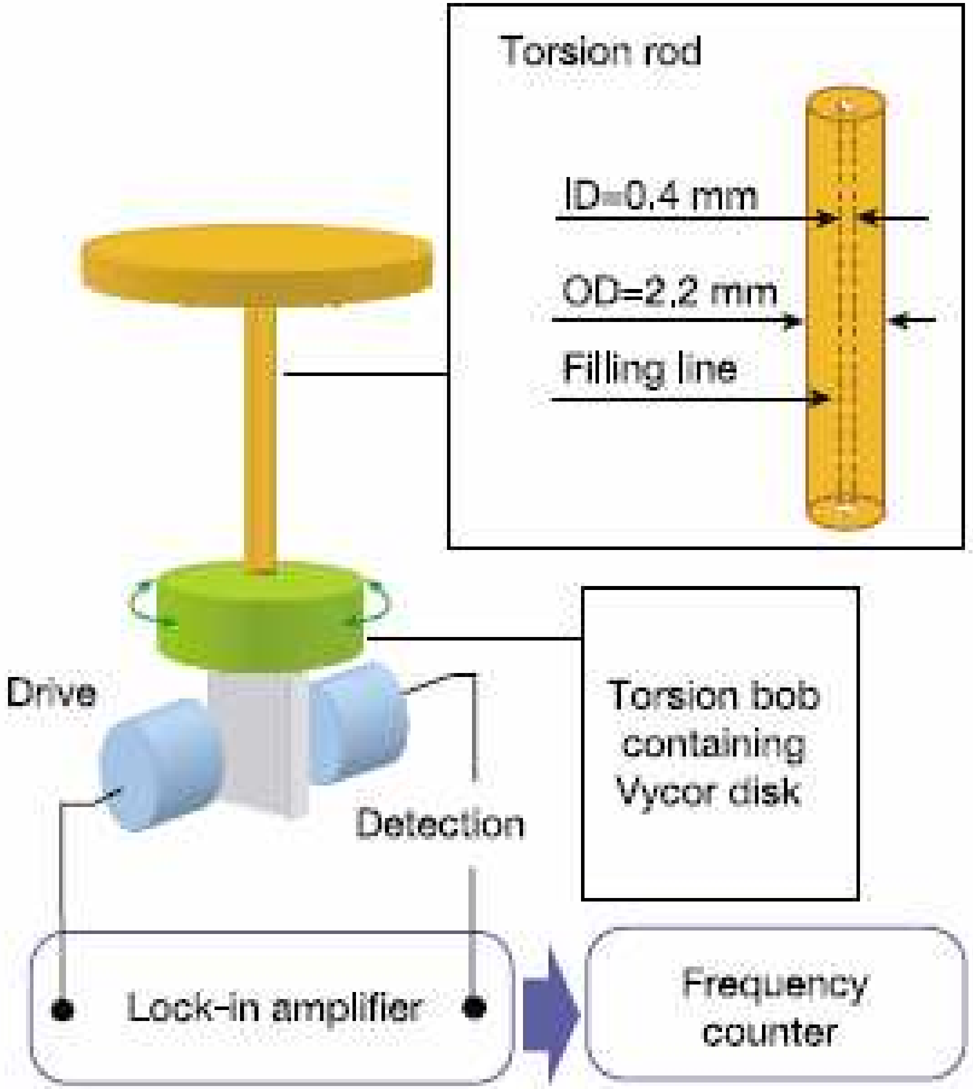
$$\frac{\rho_s}{\rho} \sim 10^{-4}$$

- Over 35 years experiments have searched for supersolids without success!

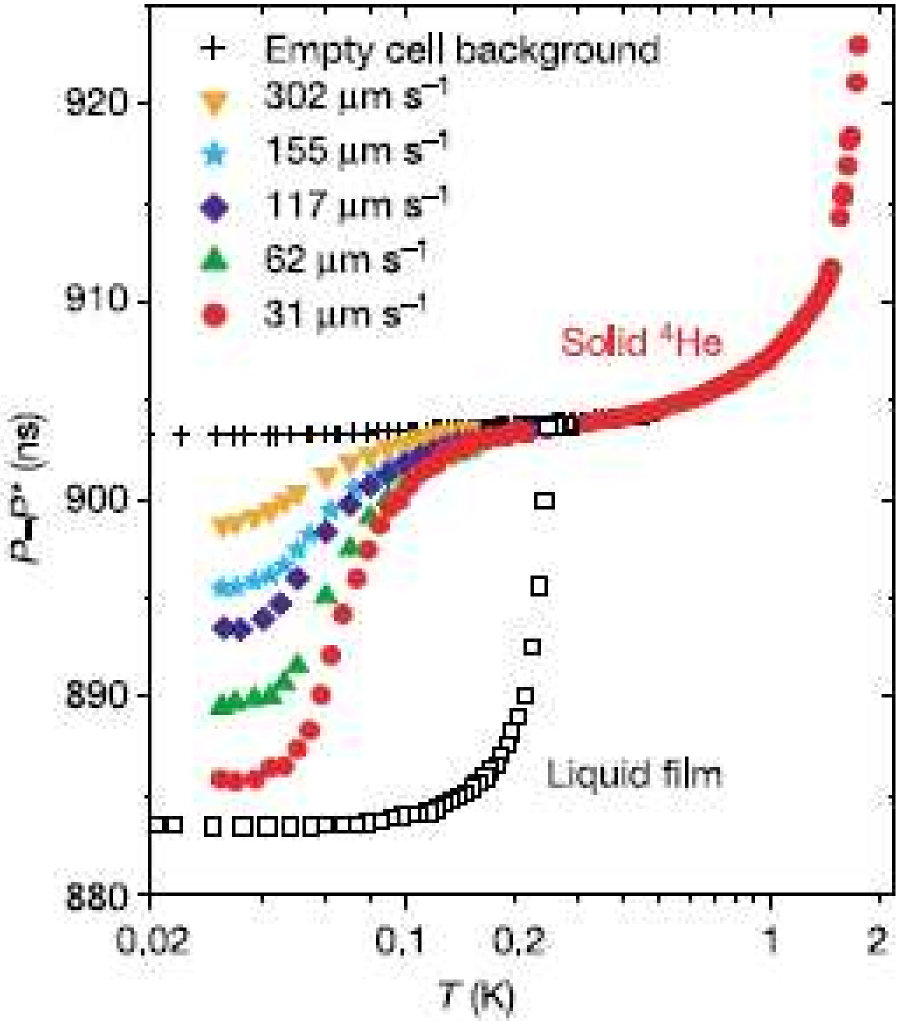
Chan-Kim experiments report observation of a supersolid

- E. Kim and M. H. W. Chan, Nature London **427**, 225 2004
- E. Kim and M. H. W. Chan, Science **305**, 1941 2005
- E. Kim and M. H. W. Chan, J. Low Temp. Phys. **138**, 859 2005
- E. Kim and M. H. W. Chan, Phys. Rev. Lett. **97**, 115302 2006

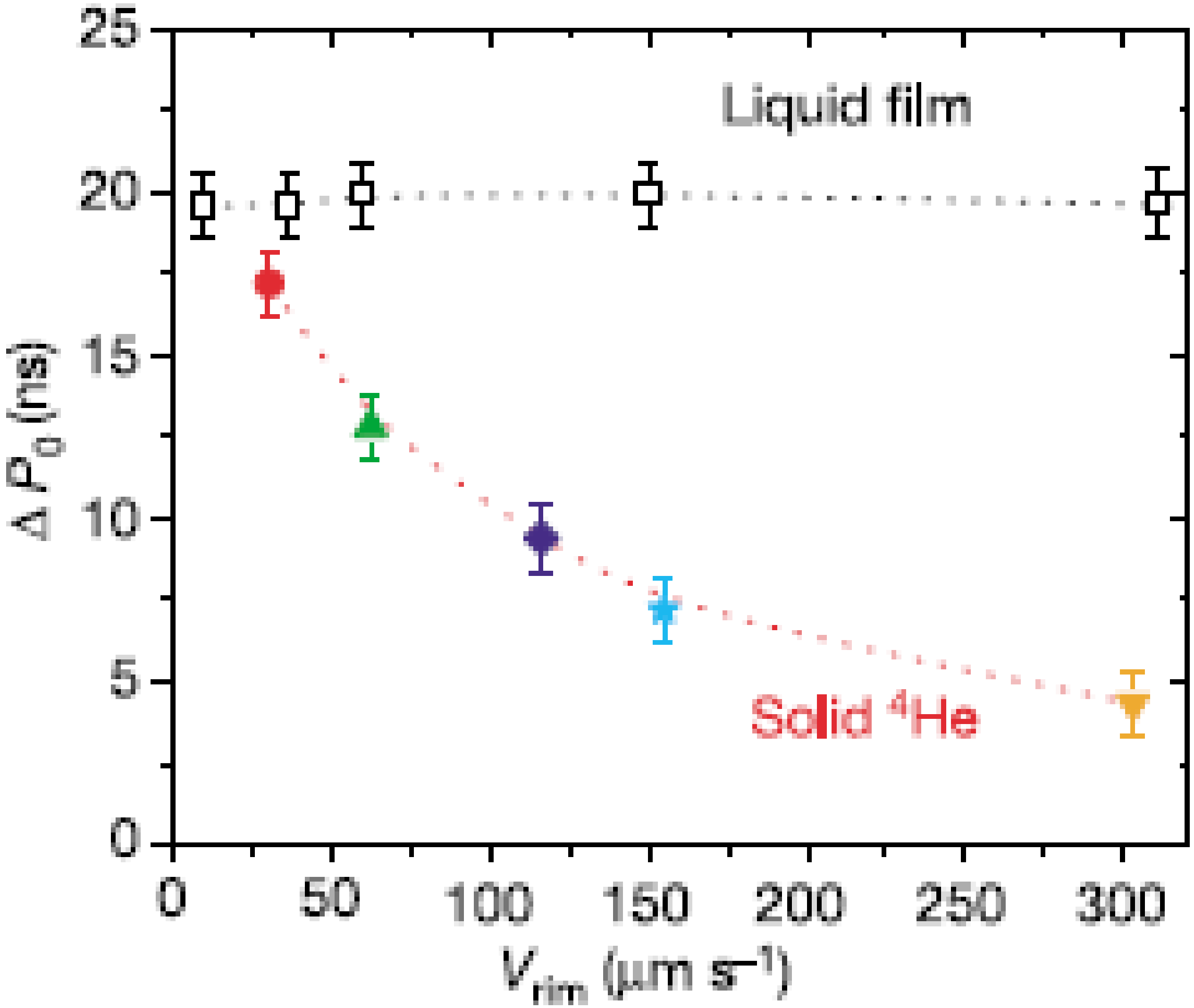
Chan-Kim torsional oscillator setup



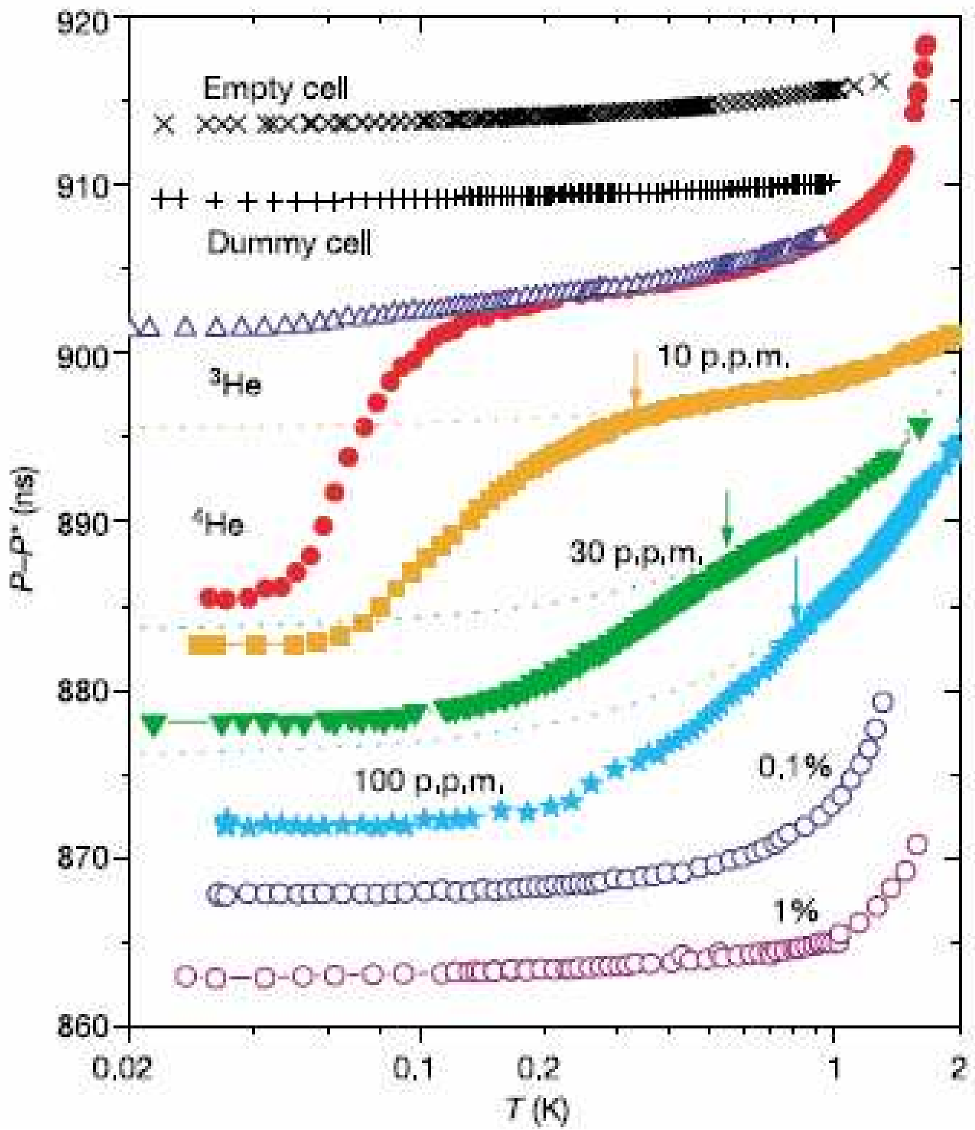
Shift in resonant period of solid ^{4}He in porous vycor glass



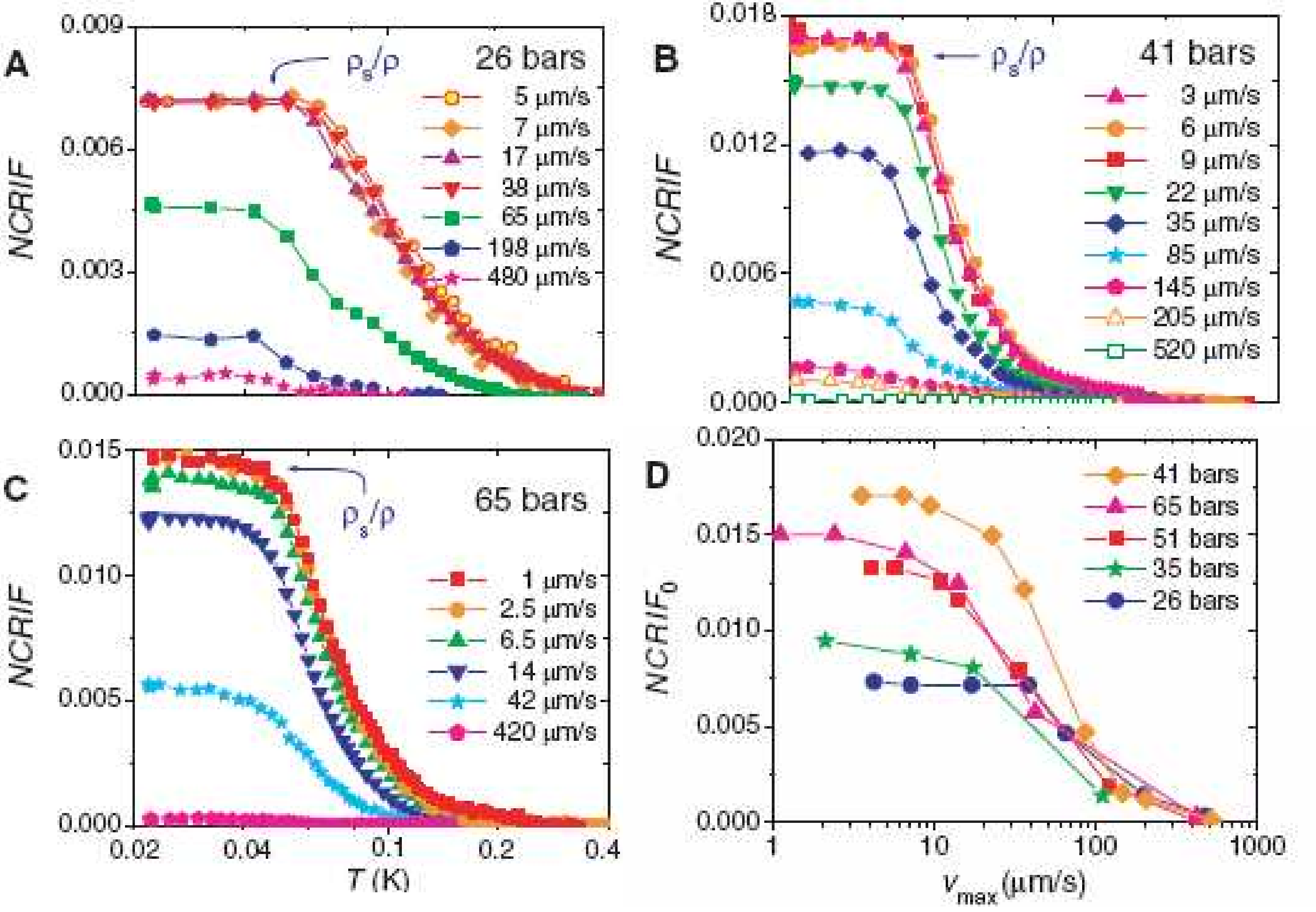
Shift in resonance period vs drive



Resonant period vs temperature of a variety of He samples

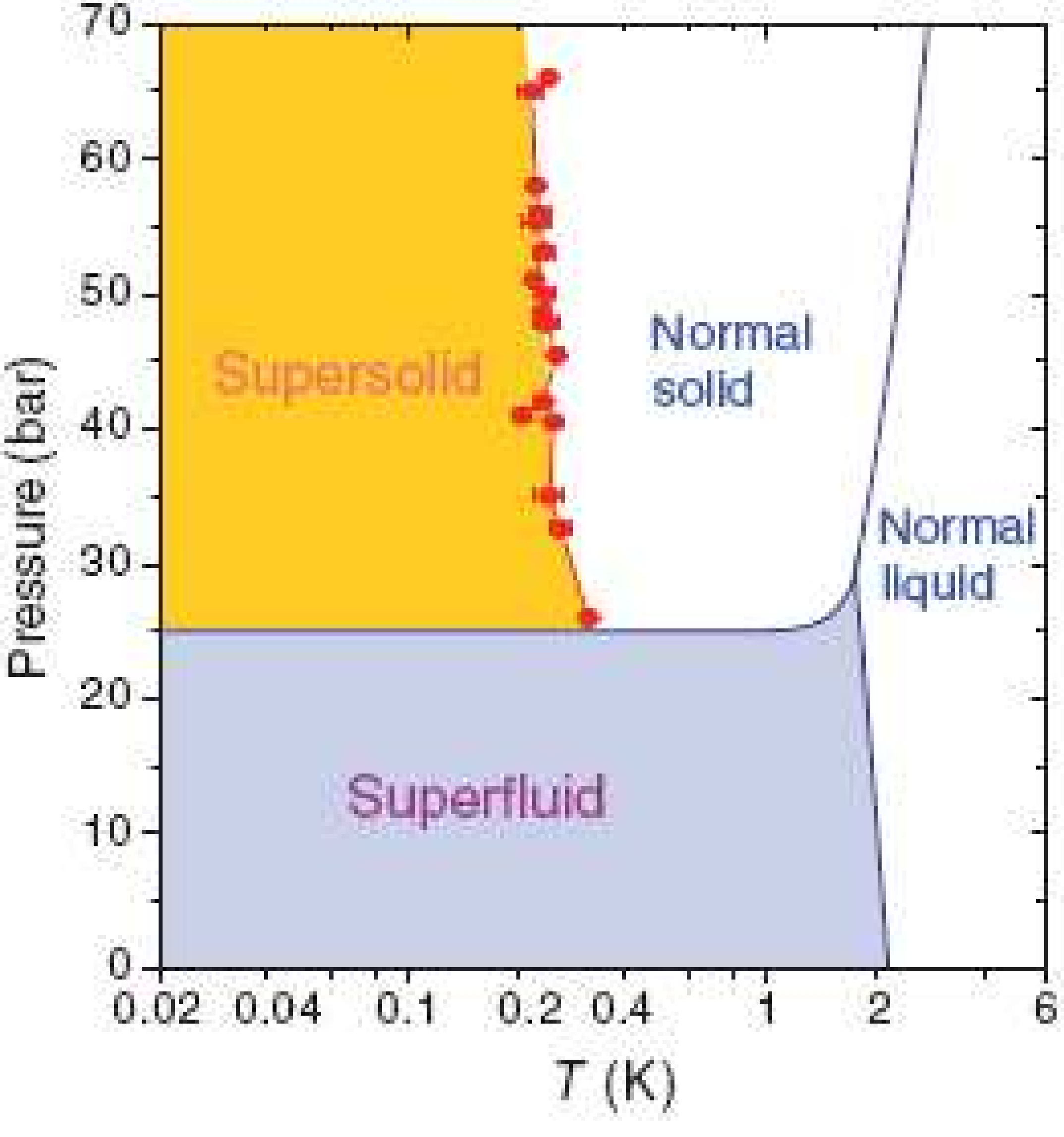


NCRIF



A
26 bars
ρ_s/ρ
5 μm/s
7 μm/s
17 μm/s
38 μm/s
65 μm/s
198 μm/s
480 μm/s
NCRIF
B
41 bars
ρ_s/ρ
3 μm/s
6 μm/s
9 μm/s
22 μm/s
35 μm/s
85 μm/s
145 μm/s
205 μm/s
520 μm/s
C
65 bars
ρ_s/ρ
1 μm/s
2.5 μm/s
6.5 μm/s
14 μm/s
42 μm/s
420 μm/s
T (K)
D
41 bars
65 bars
51 bars
35 bars
26 bars
$NCRIF_0$
v_{max} (μm/s)

Phase diagram



Summary of Chan-Kim results

- NCRI observed in solid ^{4}He in porous vycor, porous gold, and in bulk helium.
- No NCRI obtained in ^{3}He control experiment, which does not BEC
- Simplest interpretation: supersolid discovered
- No heat capacity divergence at transition

Annealing eliminates NCRI

- Confirmed the existence of a supersolid state in solid ^{4}He at temperatures below 250 mK.
- Used a torsional oscillator cell with a square cross section to insure a locking of the solid to the oscillating cell.
- NCRI can be eliminated through an annealing of the solid helium sample.

A. S. C. Rittner, J. D. Reppy, PRL **97**, 165301 (2006)

Disorder and the Supersolid State

- Torsional oscillator studies of highly disordered solid ^4He.
- Prepared our samples by rapid freezing from the normal phase of liquid ^4He, to approach glassy state.
- The supersolid signals observed for such samples are remarkably large, exceeding 20% of the entire solid helium moment of inertia.
- Together with the previous annealing study, this suggest that the supersolid state exists for the disordered or glassy state of helium and is absent in high quality crystals of solid ^4He.

A. S. C. Rittner, J. D. Reppy, PRL **98**, 175302 (2007)

Grain boundaries

- Solid atoms *delocalize* at a grain boundary. This makes a 2D He layer that can undergo a KT transition to a 2D superfluid, and perhaps explain Chan experiments?
  S. Sasaki, R. Ishiguro, F. Caupin, H.J. Maris, S. Balibar, *Science* **313,** 1098 (2006).
- Does not explain why *the same* apparent superflow response appears in various porous media such as vycor glass, porous gold and bulk helium, and at various pressures. One would expect that the density of grain boundaries would be *much greater* in porous media.

Weak First-Order Superfluid-Solid Quantum Phase Transitions

- Study superfluid-solid zero-temperature transitions in two-dimensional lattice bosons using worm-algorithm Monte Carlo simulations.
- Suggest that the superfluid-checkerboard solid and superfluid–valencebond solid transitions at half-integer filling factor are extremely weak first-order transitions
- Can in small systems be confused with continuous or high-symmetry points.

A. Kuklov, N. Prokof ’ev, B. Svistunov, PRL **93,** 230402 (2004)

Possible role of a dislocation-induced glass

- Observed cubic and linear contributions to the specific heat at *T*~200 mK are found to be substantially less than for a proposed supersolid transition with 1% superfluid fraction.
- Suggest that the low-temperature linear term in the specific heat is due to a glassy state that develops at low temperatures, caused by a distribution of tunneling systems in the crystal from small scale dislocation loops.
- Reported mass decoupling is found to be consistent with an increase in the oscillator frequency, as expected for a glasslike transition.

A. V. Balatsky, M. J. Graf, Z. Nussinov, S. A. Trugman, PRB **75**, 094201 (2007)

Alternative interpretation: Partial solidification

- The rotation period decrease is found to be also consistent with a solidification of a small liquidlike component into a low-T glass.
- Solidification by a low-temperature quench of topological defects e.g., grain boundaries or dislocations.
- Can also account the peak in the dissipation occurring near the transition point.
- Unlike the NCRI scenario, the dependence of the rotational period on external parameters, e.g., the oscillator velocity, provides an alternate interpretation of the oscillator experiments.

Z. Nussinov, A. V. Balatsky, M. J. Graf, S. A. Trugman, PRB **76**, 014530 (2007)

Analogy with vortex liquid

- **Analogy with anomalous response to vorticity in the vortex liquid phase in the pseudogap region of high-temperature superconductors.**
- **Proposal: what is observed is not supersolidity but an incompressible vortex liquid.**
- **This state is distinct from a conventional liquid in that its properties are dominated by conserved supercurrents flowing around a thermally fluctuating tangle of vortices.**

P.W. Anderson, Nature Physics 3, 160 - 162 (2007)

Possibility of supersolids in optical lattices

- Consider using excited states of cold atom optical lattices to generate *extended* range Hubbard models.
- Bosons confined to higher bands of optical lattices allow for a rich phase diagram, including the supersolid phase.
- Use Gutzwiller mean-field theory to establish parameter regime for metastable states generated by an extended Bose-Hubbard model.

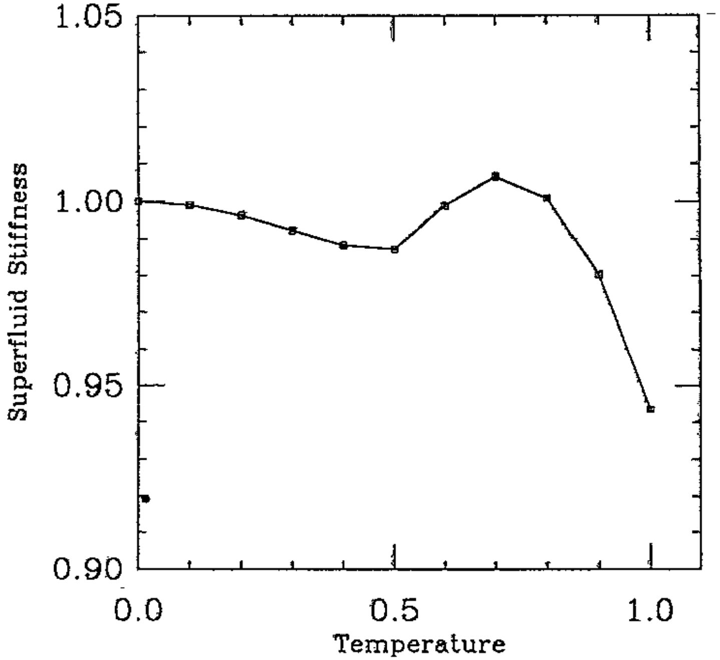
V.W. Scarola, S. Das Sarma, PRL **95,** 033003 (2005)
V.W. Scarola, E. Demler, S. Das Sarma, PRA **73**, 051601(R) (2006)

Superfluids with orientational order

- Developed a theory for novel states of ^{4}He that have both ODLRO as well as hexatic or bond orientational order.
- Both the hexatic and superfluid transitions are of Kosterlitz-Thouless type
- The SF transition is sensitive to the hexatic transition.

$$E = -\sum_{(i,j)} k_1 \cos\left(\theta(r_i) - \theta(r_j)\right) + k_2 \cos\left(\theta_6(r_i) - \theta_6(r_j)\right)$$
$$+ s\left[\cos\left(\theta(r_i) - \theta(r_j)\right) - 1\right]\left[\cos\left(\theta_6(r_i) - \theta_6(r_j)\right) - 1\right]$$

K. Mullen, H.T.C. Stoof, M. Wallin, S.M. Girvin, PRL 72, 4013 (1994)

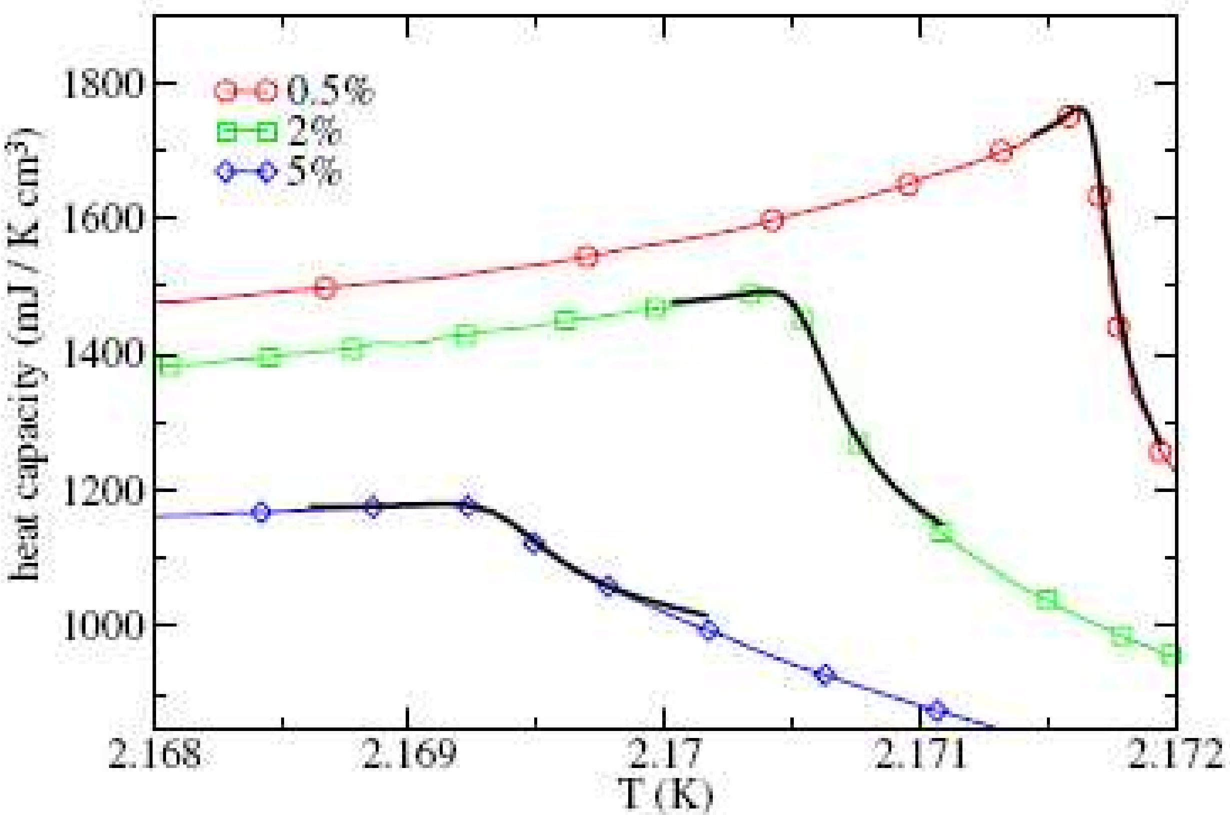


Superfluid ^{4}He in aerogel

Aerogel fractal modifies the lambda phase transition

Porous vycor glass is more like like uncorrelated disorder

$$C(T) = A_{\pm}(T)\left|T - T_c\right|^{-\alpha} + B(T)$$



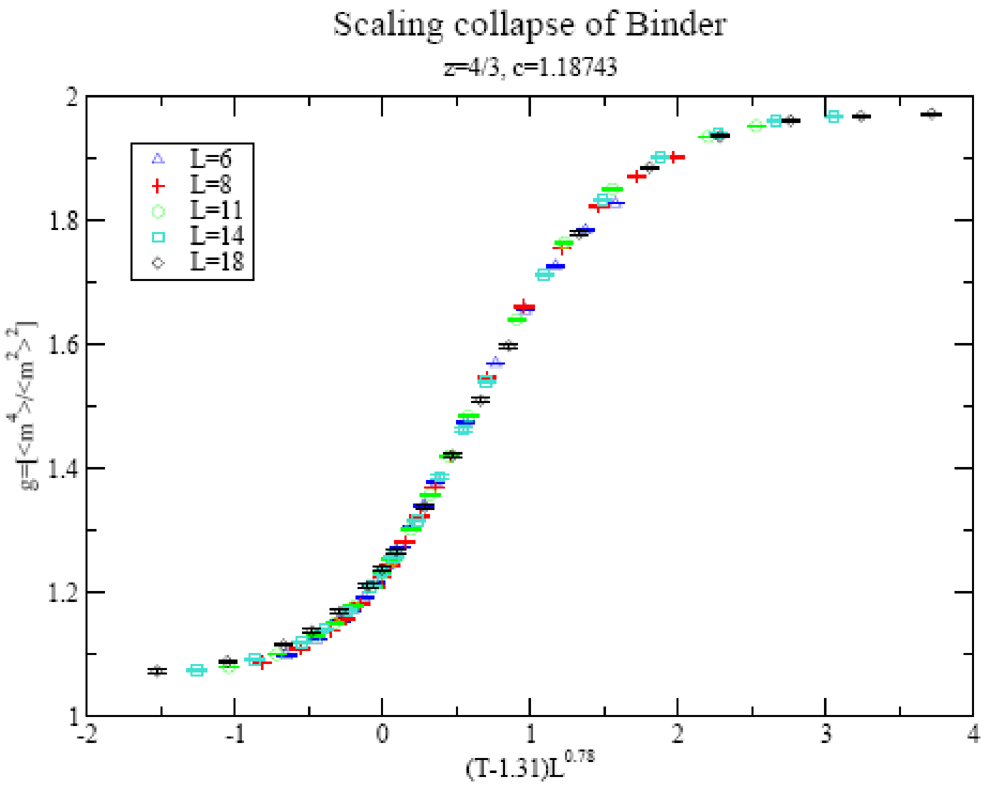
M. Nikolaou, M. Wallin, H. Weber, PRL **97,** 225702 (2006)

Quantum XY model with percolation disorder

QPT in (2+1)D XY model with porous correlated percolation disorder in the spatial couplings.
Leads to new universality class.

$$H = -\sum_{ij} J_{ij} \cos(\theta_i - \theta_j)$$

$$J_{ij} = 0,1 \quad \text{percolation cluster}$$



F. Moraeus, J. Lidmar, M. Wallin, in preparation

Supersolid quantum critical point

- Assume a 3+1 dimensional XY model for defect condensate in the ^{4}He solid
- Pure case: mean field transition $\upsilon = 0.5$
- Disorder case: porous substrate/impurities Harris criterion

$$\upsilon > \frac{2}{3} = 0.67$$

- Disordered critical point expected

Scaling of quantum critical point

- Incompressibility requirement

$$\kappa = \frac{\partial^2 f}{\partial \mu^2} \sim \xi^{z-3} \Rightarrow z = 3$$

- Quantum critical region scaling

$$T^* \sim \beta^{-1} \sim \xi^{-z} \sim (p - p_c)^{3\nu}$$

Absence of self duality

- Phase only (3+1)D XY model
- Boson integer link currents
- Vortices become topological defect sheets in 3+1 dimensions
- Vortex segment interactions

$$V(r) = \int d^4k \, \frac{e^{ik \cdot r}}{k^2} \sim \frac{1}{r^2}$$

Summary

- Possible observation of supersolid phase in torsional oscillator experiments by Chan and Kim
- Interpretation under debate
- Quantum phases of ^4He offer rich possibilities for modeling and scaling of phase transitions
- Porous media possibly modifies the supersolid phase transition