Dynamo theory and magneto-rotational instability

- seed field
- AGN outflows
- MRI driven
- primordial (decay)
- diagnostic interest (CMB)
- galactic
- LS dynamo
- helicity losses

Axel Brandenburg (Nordita)
The primordial alternative: Decay of field $\rightarrow$ growth of scale

- Starting point: EW phase transition $t=10^{-10}$ s, $B=10^{24}$ G
- Horizon scale very short: $\sim 3$ cm
- With cosmological expansion: $\sim 1$ AU
- Can field grow to larger scales?
Inverse cascade of magnetic helicity

argument due to Frisch et al. (1975)

\[ E_p + E_q = E_k \quad \text{and} \quad |H_p| + |H_q| = |H_k| \]

Initial components fully helical:

\[ 2E_p = p |H_p| \quad \text{and} \quad 2E_q = q |H_q| \]

\[ p |H_p| + q |H_q| = 2E_k \geq k |H_k| = k \left( |H_p| + |H_q| \right) \]

\[ k \leq \frac{p |H_p| + q |H_q|}{|H_p| + |H_q|} \leq \max(p, q) \quad \rightarrow k \text{ is forced to the left} \]
3-D simulations

Initial slope

$E \sim k^4$

Christensson et al.
(2001, PRE 64, 056405)
Helical decay law: Biskamp & Müller (1999)

\[ H = EL = \text{const} \]

\[ \varepsilon = \frac{U^3}{L} = \frac{E^{3/2}}{L} \]

\[ \varepsilon = -\frac{dE}{dt} \]

\[ \varepsilon = -\frac{dE}{dt} = \frac{E^{3/2}}{L} = \frac{E^{5/2}}{H} \]

\[ E \propto t^{-2/3} \]
Revised helical decay law

M. Christensson, M. Hindmarsh, A. Brandenburg: 2005, AN 326, 393

$H$ not exactly constant

$$\dot{H} = -2\eta k_H^2 H$$

Assume power law

$$k_H = k_{H0} \left( \frac{t}{t_0} \right)^{-r}$$

$H$ follows power law iff $r=1/2$; then

$$H \propto t^{-2s}$$

$$s = \eta k_{H0}^2 t_0 = \eta k_H^2 t = \left( \frac{\xi_{\text{diff}}}{\xi_H} \right)^2$$

$$E \geq |H| / \xi_I = t^{-1/2-2s}$$
All length scales scale similarly

\[ E = t^{-1/2 - 2s} \]
\[ s \approx \frac{25}{R_m} \]

\[ R(t) = -t \dot{H} / H \]
\[ Q(t) = -t \dot{E} / E \]

\[ \frac{M}{|C|} \]

\[ \frac{|H|}{M} \]

\[ \frac{s}{s_{\text{diff}}} \]

should be \( s \)

should be \( \frac{1}{2} + 2s \)
seed field

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MRI driven

galactic LS dynamo

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Accretion discs
Corona heated by MRI
Outflow (+also magn tower)

weak by comparison
Alfven and slow magnetosonic waves coupled to rotation and shear

Vertical field $B_0$

$$\dot{u}_x - 2\Omega u_y = B_{0z} b_x'$$

$$\dot{u}_x + (2 - q)\Omega u_x = B_{0z} b_y'$$

$$\dot{b}_x = B_{0z} u_x'$$

$$\dot{b}_y = B_{0z} u_y' - q\Omega b_x$$

Alfven frequency:

$$\omega_A = \nu_A k$$

Dispersion relation

$$\omega^2 - 2\omega^2 \left[ \omega_A^2 + (2 - q)\Omega^2 \right] + \omega_A^2 \left( \omega_A^2 - 2q\Omega^2 \right) = 0$$

effect of rotation, $\Omega$

effect of shear: $q$
March 23, 1965: Gemini 3

Gus Grissom & John Young: docking with Agena space craft

\[ \ddot{r}_i = -\frac{GM}{r_i^3} r_i - K(r_i - r_j) \]

Analogies:

- \( \omega_p^2 < \alpha^3 \Omega^2 \)  
  Tidal disruption of a star

- \( K < 2\Omega^2 \)  
  Space craft experiment

- \( \omega_A^2 < 2q\Omega^2 \)  
  MRI (Balbus & Hawley 1991)
Nonlinear shearing sheet simulations

Dynamo makes its own turbulence

Divergent spectrum

512^3 resolution
Vertical stratification

Brandenburg et al. (1996)

\[ \nu_{\text{turb}} = \alpha c_s H = \alpha(z) c_s H \]

\[ \rho \nu_{\text{turb}} = \rho \alpha c_s H = \alpha c_s \Sigma \approx \text{const} \]
Heating near disc boundary

$$c_v \frac{\partial T}{\partial t} = ... + \nu (\nabla u)^2 + \frac{J^2}{\rho \sigma}$$

weak z-dependence of energy density

$$\rho u^2 \approx B^2 / \mu_0$$

where

$$J = \nabla \times B / \mu_0$$

Alternative: Magnetisation from quasars?

Poynting flux

\[ B_{\text{rms}} = \sqrt{8\pi \frac{F_{\text{poynt}}}{F_{\text{kin}}} \frac{N\dot{M}_w c_s^2}{V} \Delta t} \approx 1 \mu G \]

10,000 galaxies for 1 Gyr, \(10^{44}\) erg/s each

Similar figure also for outflows from protostellar disc

seed field

primordial (decay)

AGN outflows MRI driven
galactic LS dynamo
helicity losses

Dynamo saturation
Rm dependent??
Helicity losses essential

weak by comparison
Close box, no shear: resistively limited saturation

Brandenburg & Subramanian

\( k_i \langle B^2 \rangle - k_i \langle b^2 \rangle = 0 \)

\( \langle J \cdot B \rangle + \langle j \cdot b \rangle = 0 \)

Significant field already after kinematic growth phase

followed by slow resistive adjustment

\[ \langle A \cdot B \rangle + \langle a \cdot b \rangle = 0 \]

\[ k_i^{-1} \langle B^2 \rangle - k_i^{-1} \langle b^2 \rangle = 0 \]

Connection with $\alpha$ effect:

writhe with *internal* twist as by-product

clockwise tilt (right handed)

→ left handed internal twist

\[ \alpha = -\frac{1}{3} \tau \left( \langle \omega \cdot u \rangle - \langle j \cdot b \rangle / \rho_0 \right) \]

both for thermal/magnetic buoyancy

Yousef & Brandenburg
Helicity fluxes in the presence of shear

Mean field: azimuthal average

geometry here relevant to the sun

Mean field with no helicity, e.g.

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{\omega} \times \mathbf{U})$$

Rogachevskii & Kleeorin (2003)

Subramanian & Brandenburg (2004, PRL 93, 20500)
Conclusions

- Primordial: $B^2 \sim t^{-1/2}$ (if fully helical), not $B^2 \sim t^{-2/3}$
- Outflows: via MRI-heated corona
- Dynamo: $j \cdot b$ saturation
  - even for $WxJ$ effect
  - (only shear, no stratification)
- Helical outflows necessary
- Possible for shear flow

$10^{46} \text{ Mx}^2/\text{cycle}$
(for the sun)