2D compressibility of surface states on 3D topological insulators

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Why compressibility?

- We consider the topological surface states of 3D crystals such as Bi$_2$Te$_3$ and Bi$_2$Se$_3$.
- Many probes (e.g. transport, optical measurements) sample bulk and surface states.
- Surface measurement desireable to characterize topological properties.
- Compressibility is pure ground state property. (Interacting ground state!)
Why compressibility?

- We consider the topological surface states of 3D crystals such as Bi$_2$Te$_3$ and Bi$_2$Se$_3$.
- Many probes (e.g. transport, optical measurements) sample bulk and surface states.
- Surface measurement desirable to characterize topological properties.
- Compressibility is pure ground state property. *(Interacting ground state!)*
- Use single electron transistor microscopy (SET).
- Intermediate sampling size averages over disorder.
- We calculate $\frac{d\mu}{dn}$, where $\mu$ is chemical potential, $n$ is extrinsic carrier density.

$$\frac{1}{K} = n^2 \frac{d\mu}{dn}$$

Figure: Martin et al., Nat. Phys. 4, 144 (2008). Schematic of SET and data for monolayer graphene.
**Single particle theory**

\[
H^l = \hbar v_F (k_y \sigma_x - k_x \sigma_y)
\]

Band velocity \(v_F\).

\[
\frac{d\mu^l}{dn} = \hbar v_F \sqrt{\pi |n|}
\]

\[
H^q = \hbar v_F (k_y \sigma_x - k_x \sigma_y) + \frac{\hbar^2 k^2}{2m^*}
\]

Includes band curvature seen in ARPES, \(m^*\).

\[
\frac{d\mu^q}{dn} = \frac{2\pi \text{sgn}(n) \hbar^2}{m^*} + \hbar v_F \sqrt{\pi |n|}
\]

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**Figure:** Xia et al., Nat. Phys. 5, 395 (2009).
Exchange self-energy

- We include electron–electron interactions via the exchange contribution to the self-energy. (Dominant term due to high dielectric constant.)

\[
\Sigma_{\text{ex},\lambda} = -\frac{1}{8\pi^2} \int d^2 q \sum_{\lambda'} V_C(q) f_{\lambda'}(k + q) \left[ 1 + \lambda' \cos(\theta_{k+q} - \theta_k) \right]
\]

- \( V_C(q) = 2\pi e^2/(\kappa|q|) \) is the unscreened Coulomb interaction.
- This equation is true for \( H^l \) and \( H^q \) since band structure only appears in \( f_\lambda(k) \), the Fermi function.

- The \( q \) integration contains an ultra-violet divergence so we define a cutoff \( k_c \).

![Graph showing exchange self-energy](image-url)
\( \frac{d\mu}{dn} \) with interactions

- The interacting \( \frac{d\mu}{dn} \) is given by

\[
\mu_{HF} = \mu + \Sigma_{ex}
\]

- With \( \epsilon = 1 \) and small \( k_c \), \( \frac{d\mu_{HF}}{dn} \) may be negative.
- Increase or decrease in \( \frac{d\mu_{HF}}{dn} \) depending on \( k_c \).
- Linear increase in \( \frac{d\mu_{HF}}{dn} \) with \( \log(k_c a) \).

This reveals renormalization of the Fermi velocity

\[
v_{F,ex} = \frac{1}{\hbar} \frac{d\mu_{HF}}{dk} \sim \left( 1 + r_s \log \frac{k_c}{k_F} \right)
\]
Quadratic correction

- Turning point in valence band not present in ARPES data.
- Can define $k_c = k_0 = v_F m^*/\hbar$
- Or use ‘turned’ band structure

$$\epsilon_{-k}^q = -\frac{\hbar^2 k^2}{2m^*} - \hbar v_F k - v_F^2 m^*$$

![Graphical representation of quadratic correction](image)
Realistic dielectric environment

- $\kappa = 20$ reduced interaction strength.
- Only very small deviation from single-particle prediction.

**Figure**: Best fit parameters: $m^* = 0.417m_e$, $v_F = 1.00 \times 10^6$ ms$^{-1}$.

- ARPES data from Zhu et al..
- Compute $\frac{d\mu}{dn}$ numerically from bandstructure.
- Do best fit to our theory.
Summary

- Scanning SET spectroscopy will be a valuable tool for investigating surface states of 3D TIs.

- $d\mu/dn$ largely unaffected by interactions because dielectric environment is very strong.

- Careful analysis may reveal
  - Fermi velocity renormalization.
  - Information about how to set $k_c$.

- Asymmetry between electron and hole bands will reveal curvature of bands, $m^*$.