Energy correlations at conformal collider

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Energy flow correlations in QCD

Conformal collider for $\mathcal{N} = 4$ SYM

Why conventional approach is not efficient

Generalized optical theorem

Energy flow correlations in $\mathcal{N} = 4$ SYM

From $\mathcal{N} = 4$ SYM to QCD
$e^+e^- \text{ annihilation in QCD}$

- PETRA (1978-1986) and LEP (1989-2010)

- A virtual photon or $Z^0$—boson decay into quarks and gluons that undergo a hadronization process into hadrons

- Final states can be described using the class of *infrared finite* observables (event shapes):
  - energy-energy correlations (EEC), thrust, heavy mass, . . .

- Can be computed in perturbative QCD, hadronisation corrections are ‘small’ at high energy
Energy-energy correlation

✓ Function of the angle $0 \leq \chi \leq \pi$ between detected particles
  
  [Basham,Brown,Ellis,Love’78]

\[
EEC(\chi) = \sum_{a,b} \int d\sigma_{a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos \theta_{ab} - \cos \chi)
\]

Total energy $\sum_a E_a = Q$

✓ One of the best studied event shapes

✓ The final states are dominated by two-jet events

✓ Current status (1978 – today):
  
  × Very precise experimental data
  
  × Slow progress on the theory side

\[
EEC(\chi) = a_s(Q) A(\chi) + a_s^2(Q) B(\chi) + O(a_s^3)
\]

Basham et al 1978 Dixon et al 2018

✓ Final goal: develop more efficient method to computing EEC

DELPHI data

N_{jet} = 2

N_{jet} = 3

N_{jet} = 4

N_{jet} = 5

Sherpa1.2.1

Energy-energy correlation, EEC

MC/data
Conformal collider: $e^+ e^-$ annihilation in $\mathcal{N} = 4$ SYM

Use $\mathcal{N} = 4$ SYM for developing new approaches to computing physical observables in QCD

Introduce an analog of the QCD electromagnetic current: the stress-energy supermultiplet

$J = \left\{ O_{20}', \varepsilon^\mu J_{R, \mu}, \varepsilon^{\mu\nu} T_{\mu\nu} \right\}$

1/2-BPS operator $\quad R-$current $\quad$ stress-energy tensor

The final state contains an arbitrary number of scalars ($s$), gauginos ($q$) and gauge fields ($g$)

$$\int d^4x \ e^{iQx} J(x)|0\rangle = |ss\rangle + |ssg\rangle + |sqq\rangle + \ldots$$

Energy-energy correlation in $\mathcal{N} = 4$ SYM

$$\text{EEC}(\chi) = \sum_{a,b=s,q,g} \int d\sigma_{a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos \theta_{ab} - \cos \chi)$$
Conventional approach

✔ EEC as a weighted cross-section

\[
\text{EEC}(\chi) = \sum_{a, b, X} \int d\text{LIPS} |A_{a+b+X}|^2 \frac{E_a E_b}{Q^2} \delta(\cos \chi - \cos \theta_{ab})
\]

The amplitude of creation of the final state \(|a, b, X = \text{everything}\rangle\)

\[
A_{a+b+X} = \int d^4x e^{iQx} \langle a, b, X | J(x) | 0 \rangle
\]

✔ New approach: EEC can be computed from \textit{correlation functions of energy flow operators}

✗ presence of infrared divergences in transition amplitudes \(A_{a+b+X}\)

✗ integration over the Lorentz invariant phase space of the final states \(d\text{LIPS}\)

✗ necessity for summation over all final states \(\sum_X\)

✗ no analytical results beyond one loop

Main disadvantages:
EEC from correlation functions

☑ Total cross section from the optical theorem

\[ \sigma_{\text{tot}}(q) = \sum_X (2\pi)^4 \delta^{(4)}(Q - p_X) |\mathcal{A}_{J \rightarrow X}|^2 \]

\[ = \int d^4 x \ e^{iQx} \sum_X \langle 0|J^\dagger (0)|X\rangle \ e^{-i p_X X} \langle X|J(0)|0\rangle \]

\[ = \int d^4 x \ e^{iQx} \langle 0|J^\dagger (x)J(0)|0\rangle = \frac{1}{16\pi} (N^2 - 1) \theta(Q^0) \theta(Q^2) \]

☑ Generalization to EEC

\[ \text{EEC} \sim \sum_X \langle 0|J^\dagger (x)|X\rangle \ w(X) \langle X|J(0)|0\rangle = \langle 0|J^\dagger (x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) J(0)|0\rangle \]

☑ Energy flow operator

[Sveshnikov, Tkachov], [GK, Oderda, Sterman]

\[ \mathcal{E}(\vec{n})|X\rangle = \sum_a E_a \delta^{(2)}(\Omega_{\vec{p}_a} - \Omega_{\vec{n}})|X\rangle \]

Relation to the energy-momentum tensor in $\mathcal{N} = 4$ SYM

\[ \mathcal{E}(\vec{n}) = \int_0^\infty dt \ \lim_{r \rightarrow \infty} r^2 \ \vec{n}^i T_{0i}(t, r\vec{n}) \]
EEC from correlation functions II

✔ Energy flow correlations

\[ \text{EEC}(\chi) = \int d^4x \, e^{iQx} \langle 0 | J^\dagger(x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) J(0) | 0 \rangle \]

Energy flow in the direction of \( \vec{n}_1 \) and \( \vec{n}_2 \) with the relative angle \( \chi \)

✔ Multi-fold integral of Wightman 4pt function

\[ \text{EEC} \sim \int d^4x \, e^{iQx} \int_0^\infty dt_1 dt_2 \lim_{r_i \to \infty} r_1^2 r_2^2 \langle 0 | J^\dagger(x) T_0(\vec{n}_1(x_1)) T_0(\vec{n}_2(x_2)) J(0) | 0 \rangle \]

\[ x_i = (t, r \vec{n}_i) \]

✗ Compute corr. function \( \langle J^\dagger(x) T(x_1) T(x_2) J(0) \rangle \) in Euclid

✗ Continue to Minkowski with Wightman prescription

✗ Take detector limit + perform Fourier

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Correlation functions in $\mathcal{N} = 4$ SYM

- Correlation functions of $J = \{O_{20}', J_{R,\mu}, T_{\mu\nu}\}$ in the stress-energy multiplet are determined by the same scalar function

\[
\langle O(x_1)O(x_2)O(x_3)O(x_4) \rangle_E = \frac{1}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2} \Phi(u, v; a)
\]

\[
\langle J(x_1)T(x_2)T(x_3)J(x_4) \rangle_E = \frac{1}{(x_{12}^2 x_{23}^2 x_{34}^2)^2} P(\partial_u, \partial_v) \Phi(u, v; a)
\]

Cross-ratios

\[
u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{23}^2 x_{41}^2}{x_{13}^2 x_{24}^2}
\]

- Universal function at weak coupling $a = g^2_{YM} N_c/(4\pi^2)$

\[
\Phi(u, v) = a \Phi^{(1)}(u, v) + a^2 \left( \frac{1}{2} (1 + u + v) \left[ \Phi^{(1)}(u, v) \right]^2 + 2 \left[ \Phi^{(2)}(u, v) + \frac{1}{u} \Phi^{(2)}(v/u, 1/u) + \frac{1}{v} \Phi^{(2)}(1/v, u/v) \right] \right) + O(a^3)
\]

$\Phi^{(1)}(u, v)$ 'box' integral, $\Phi^{(2)}(u, v)$ 'double' box integral

- AdS/CFT correspondence predicts $\Phi(u, v)$ at strong coupling
All-loop prediction for EEC

Master formula

\[
\text{EEC}(\chi) = \frac{1}{4z^2(1-z)} \int_{-\delta - i\infty}^{-\delta + i\infty} \frac{d j_1 d j_2}{(2\pi i)^2} \frac{M(j_1, j_2; a) K(j_1, j_2)}{(1-z)^{j_1+j_2}}
\]

- The dependence on the angle \( \chi \) enters through

\[
z = (1 - \cos \chi)/2, \quad 0 < z < 1
\]

- Detector function is independent on the coupling

\[
K(j_1, j_2) = \frac{2 \Gamma(1 - j_1 - j_2)}{\Gamma(j_1 + j_2)\Gamma(1 - j_1)\Gamma(1 - j_2)^2}
\]

- The dependence on the coupling constant resides in the Mellin amplitude

\[
\Phi(u, v; a) = \int_{-\delta - i\infty}^{-\delta + i\infty} \frac{d j_1 d j_2}{(2\pi i)^2} M(j_1, j_2; a) u^{j_1} v^{j_2}
\]

- The Mellin amplitude \( M(j_1, j_2; a) \) is known in \( \mathcal{N} = 4 \) SYM at weak and at strong coupling
EEC at weak coupling

$$\text{EEC}_{\mathcal{N}=4} = \frac{1}{4z^2(1-z)} \left\{ a F_1(z) + a^2 F_2(z) + a^3 F_3(z) + O(a^4) \right\}, \quad z = \frac{1}{2}(1 - \cos \chi)$$

- **Leading order** \( F_1(z) = -\ln(1 - z) \)

- **Next-to-leading order**

$$F_2(z) = (1-z)(4\sqrt{z} \left[ \text{Li}_2\left(-\sqrt{z}\right) - \text{Li}_2\left(\sqrt{z}\right) + \frac{1}{2} \ln z \ln\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right) \right] + (1+z)\left[2\text{Li}_2(z) + \ln^2(1-z)\right] + 2 \ln(1-z) \ln\left(\frac{z}{1-z}\right) + \frac{2}{3} \pi^2)$$

$$+ (1-z)(1+2z) \left[ \ln^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right) \ln\left(\frac{1-z}{z}\right) - 8\text{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}+1}\right) - 8\text{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}-1}\right) \right] - 4(z-4)\text{Li}_3(z) + 6(3+3z-4z^2)\text{Li}_3\left(\frac{z}{z-1}\right)$$

$$- 2z(1+4z)\zeta_3 + 2\left[ (3-4z)z \ln z + 2(2z^2 - z - 2) \ln(1-z) \right] \text{Li}_2(z) + \frac{1}{3} \ln^2(1-z) \left[ 4(3z^2 - 2z - 1) \ln(1-z) + 3(3-4z)z \ln z \right]$$

$$+ \frac{\pi^2}{3} \left[ 2z^2 \ln z - (2z^2 + z - 2) \ln(1-z) \right]$$

- **Next-to-next-to-leading order**

$$F_3(z) = \text{a sum of harmonic polylogarithms + a two-fold elliptic integral}$$

- **Large logarithmically enhanced corrections for** \( z \to 0 \) (small angle) and \( z \to 1 \) (back-to-back region)

[Belitsky, Hohenegger, GK, Sokatchev, Zhiboedov'2013]

[Henn, Sokatchev, Yan, Zhiboedov'2019]
From weak to strong coupling

- At weak coupling $\text{EEC}_{\mathcal{N}=4}$ has a shape which is remarkably similar to the one in QCD
- Going from one to two loops, EEC flattens
- This agrees with strong coupling prediction for EEC in planar $\mathcal{N} = 4$ SYM

$$\text{EEC}_{\mathcal{N}=4} \xrightarrow{a \to \infty} \frac{1}{2} \left[ 1 + a^{-1} (1 - 6z(1 - z)) + O(a^{-3/2}) \right]$$

No jets at strong coupling!

What is a manifestation of integrability of $\mathcal{N} = 4$ SYM?
End-point asymptotics

✔ Small angle correlations $\chi \to 0$ (or $z \sim \chi^2 \to 0$): calorimeters measure nearly collinear particles

\[
\text{EEC } z \xrightarrow{\to} 0 \quad \frac{a}{4z} \left[ 1 + a \left( \ln z - \frac{1}{2} \zeta_3 + \zeta_2 - 3 \right) \right]
\]

✗ Resummation of leading log's $a (a \ln z)^k$ using the “jet calculus” [Konishi, Ukawa, Veneziano]

\[
\text{EEC } z \xrightarrow{\to} 0 \quad \frac{a}{4z} \int_0^1 dx \: x^2 \left\{ \text{ fragmentation function } D(x, Q^2 z) \right\} = \frac{a}{4} z^{-1 + \gamma(3)/2}
\]

$\gamma(3) = 2a + O(a^2)$ – twist-2 anom. dimension of spin $S = 3$

✔ EEC in the back-to-back kinematics $\chi \to \pi$ (or $y \equiv 1 - z \sim (\pi - \chi)^2 \to 0$)

\[
\text{EEC } z \xrightarrow{\to} 1 \quad \frac{1}{4y} \left\{ a \ln(1/y) - \frac{a^2}{2} \left[ \ln^3(1/y) + \frac{\pi^2}{2} \ln(1/y) \right] \right\}
\]

✗ Large (Sudakov) corrections $a^k (\ln y)^n$ come from the emission of soft and collinear particles

✗ Can be resummed to all orders in the coupling [Collins, Soper]
Back-to-back region

Unitary diagram describing a two-jet cross-section

\[
\text{EEC} \sim \langle J^{\mu_1}(x_1)T_{\mu_2\nu_2}(x_2)T_{\mu_3\nu_3}(x_3)J^{\mu_4}(x_4) \rangle
\]

The ‘source operators’ are at the points \(x_1, x_4\), the ‘calorimeter operators’ are at \(x_2, x_3\)

The four operators are light-like separated \(x_{12}^2, x_{13}^2, x_{24}^2, x_{34}^2 \to 0\)

EEC in the back-to-back region = Light-like limit of the correlation function

\[
\langle O(x_1)O(x_2)O(x_3)O(x_4) \rangle = \frac{1}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2} \Phi(u, v; a)
\]

Cross-ratios \(u, v \to 0\)

\(\Phi(u, v; a)\) receives large double-log corrections \((a \ln u \ln v)^{\ell}\)
Light-like limit of the correlation function

OPE expansion \((x_i - x_{i+1})^2 \to 0\)

\[
O(x_i)O(x_{i+1}) \sim \sum_{\Delta,S} \frac{C_S}{(x_{i,i+1}^2)^{(\Delta-S)/2}} O_S(x_i)
\]

The leading contribution in all channels comes from the twist-2 operators with large spin \(S \gg 1\)

\[
\Delta - S = 2 + \gamma_S(a), \quad \gamma_S = 2\Gamma_{\text{cusp}}(a) \ln S + \Gamma(a)
\]

\[
C_S(a) = H(a) e^{-\Gamma(a) \ln S} 2^{-\gamma_S(a)} \Gamma^2 \left(1 - \frac{1}{2} \gamma_S(a) \right)
\]

\(\Gamma_{\text{cusp}}(a)\) and \(\Gamma(a)\) are known for any coupling from integrability; \(H(a)\) is known at three loops

\[
\Phi(u, v) \sim \sum_{S \gg 1} C_S u^{\gamma_S/2} g_S(v) \text{ conformal block}
\]

Leading asymptotics of the correlation function for \(u, v \to 0\) \cite{Alday, Eden, GK, Maldacena, Sokatchev} \cite{Alday, Bissi}

\[
\Phi(u, v) = H(a) \int_0^\infty \frac{dy_1}{y_1} \int_0^\infty \frac{dy_2}{y_2} e^{-\frac{1}{2} \Gamma_{\text{cusp}}(a) \ln(u/y_1) \ln(v/y_2) + \frac{1}{2} \Gamma(a) \ln(uv/(y_1 y_2))} f(y_1) f(y_2)
\]

The function \(f(y) = 2y K_0(2\sqrt{y})\) describes the large spin limit of the conformal block

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EEC in the back-to-back region

Prediction for the EEC in the back-to-back region \( \delta = (\pi - \chi)^2 \to 0 \)

\[
\text{EEC}(\chi) = \frac{H(a)}{8\delta} \int_0^\infty dy J_0(\sqrt{y}) \exp \left[ -\frac{1}{2} \Gamma_{\text{cusp}}(a) \ln^2(y/\delta) - \Gamma(a) \ln(y/\delta) \right]
\]

Weak-coupling expansion (with \( L = \ln(1/\delta) \gg 1 \))

\[
\text{EEC} = \frac{1}{4\delta} \left\{ aL + a^2 \left( -\frac{L^3}{2} - \frac{\pi^2 L}{4} + \frac{\zeta_3}{2} \right) + a^3 \left( \frac{L^5}{8} + \frac{\pi^2 L^3}{6} - \frac{11\zeta_3 L^2}{4} + \frac{61\pi^4 L}{720} - \frac{\pi^2 \zeta_3}{3} - \frac{7\zeta_5}{2} \right) \right.
\]
\[
+ a^4 \left[ -\frac{L^7}{48} - \frac{5\pi^2 L^5}{96} + \frac{95\zeta_3 L^4}{48} - \frac{29\pi^4 L^3}{480} + \left( \frac{67\pi^2 \zeta_3}{48} + \frac{69\zeta_5}{4} \right) L^2 \right.
\]
\[
\left. - \left( \frac{97\zeta_3^2}{8} + \frac{367\pi^6}{12096} \right) L + \frac{187\pi^4 \zeta_3}{1440} + \frac{95\pi^2 \zeta_5}{48} + \frac{785\zeta_7}{32} \right] + O(a^5) \}
\]

Homogenous weight property: \( w(L) = 1, w(\pi) = 1, w(\zeta_n) = n \quad \Rightarrow \quad w(\text{EEC}|_{\alpha^\ell}) = 2\ell + 1 \)

Relation to QCD

\[
\text{EEC}_{\text{QCD}}(\chi \to \pi) = \text{EEC}_{\mathcal{N}=4}(\chi \to \pi) + \text{lower weight terms}
\]

\( \mathcal{N} = 4 \) captures the most complicated part of the QCD result
EEC at small angles

Correlation between the particles within the same jet

\[
\text{EEC} \sim \langle J_{\mu_1}(x_1) T_{\mu_2\nu_2}(x_2) T_{\mu_3\nu_3}(x_3) J_{\mu_4}(x_4) \rangle
\]

Small angle limit \((x_2 - x_3)^2 \to 0:\)

\[
u' = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = 1/v \to 0, \quad v' = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = u/v \to 1
\]

The leading contribution comes from the twist-six operator

\[
\Phi(u, v) = \sum_{S=0,2,4,...} C_{S+2}(a)(u')^{3+\gamma_{S+2}(a)/2} g_S(v') + \ldots
\]

EEC at small angle \(z = \chi^2/4 \to 0\)

\[
\text{EEC} = z^{-1+\gamma_1(a)/2} \frac{C_1(a) \Gamma(3 + \gamma_1(a))}{4\Gamma^3(2 + \gamma_1(a)/2) \Gamma(-1 - \gamma_1(a)/2)}
\]

Depends on the twist-2 conformal data \(C_S\) and \(\gamma_S\) for small spin \(S = 1\)

\[
\text{EEC}_{QCD}(\chi \to 0) = \text{EEC}_{N=4}(\chi \to 0) + \text{lower weight terms}
\]
Conclusions and open questions

✔️ Energy correlations are good/nontrivial physical observables in $\mathcal{N} = 4$ SYM

✔️ Relation to energy-energy correlations in QCD (most complicated part)?

✔️ Interpolation between weak and strong coupling? what is the manifestation of integrability?

✔️ Other proposals for ‘good’ observables?