Asymmetric shockwave collisions and pre-hydro modeling of heavy ion collisions

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based on work with Sebastian Waeber, Andreas Rabenstein, and Andreas Schäfer
arXiv:1906.05086

Holographic QCD, Nordita, Stockholm, July 23, 2019
outline

- Motivation
- Symmetric planar shock collisions: review
- Asymmetric planar shock collisions
- Pre-hydro modeling: future hopes
heavy ion collisions

- ultra-relativistic: energy/nucleon $\gg \Lambda_{QCD}$

\[
\begin{align*}
\text{time} & \\
0 & \quad \text{parton liberation} \\
\tau_{\text{hydro}} & \\
\tau_{\text{had}} & \quad \text{hydrodynamization (aka “thermalization”)} \\
\tau_{\text{freeze}} & \quad \text{hadronization} \\
& \quad \text{hadron gas} \\
& \quad \text{freeze-out} \\
\end{align*}
\]

weakly coupled “glasma” = high multiplicity, small-$x$

$$$ = not weakly coupled, pre-hydro, far-from-equilibrium

strongly coupled QGP = near-ideal fluid
hydrodynamic modeling

• Need initial data at time $t_0$: energy & momentum density $\varepsilon(x, t_{\text{hydro}})$, $\pi(x, t_{\text{hydro}})$

• Choices:
  
  • Pretend pre-hydro evolution is irrelevant, $\varepsilon(x, t_{\text{hydro}}) = \varepsilon(x, 0)$, $\pi(x, t_{\text{hydro}}) = \pi(x, 0)$
  
  • Ad-hoc inclusion of, e.g., “initial” flow at time $t_{\text{hydro}}$

  • Use holography, pretend $\mathcal{N} = 4$ SYM $\approx$ QCD
colliding planar shocks: symmetric collisions

P. Chesler and L. Yaffe, 1011.3562
J. Casalderry-Solano, M. Heller, D. Mateos, W. van der Schee, 1305.4919, 1312.2956
P. Chesler, N. Kilbertus, W. van der Schee, 1507.02548
symmetric collisions

- Incoming shocks: Gaussian energy density profiles, width $w$:

  $$h(z) \equiv \mu^3 (2\pi w^2)^{-1/2} e^{-\frac{1}{2}z^2/w^2}$$

- Single shock metric (Fefferman-Graham coordinates):

  $$ds^2 = \tilde{\rho}^{-2} \left(-d\tilde{x}_+ d\tilde{x}_- + d\tilde{x}_\perp^2 + d\rho^2\right) + \tilde{\rho}^2 h(\tilde{x}_\pm) d\tilde{x}_\mp^2 \quad \tilde{x}_\pm \equiv \tilde{t} \pm \tilde{z}$$

- Initial data: superpose two counter-propagating shocks, transform to null infalling coordinates

- Time evolution: use characteristic formulation of general relativity, spectral methods for discretization

  - Details: horizon finding, Chebyshev & Fourier grids, domain decomposition, filtering, RK4 time stepping, …
symmetric collisions

wide shocks, \( w = 0.375/\mu \)

Figure 2. Rescaled energy density \( \frac{T^{00}}{\mu^4} \) as a function of time \( \mu t \) and longitudinal position \( \mu z \) for Gaussian shock profiles. Top figure: wide shocks with \( w = 5 w_0 \). Bottom figure: narrow shocks with \( w = w_0 \). In both plots, the shocks approach each other along the \( \mu z \) axis and collide at \( \mu z = 0 \) at time \( \mu t = 0 \). The collisions produce debris that fills the forward light cone. Nevertheless, there are clear qualitative differences in the energy density near the forward light cone. For \( w = w_0 \) there are clear post-collision remnants of the initial shocks propagating on the light cone. These remnants decay with time like \( \mu t^p \) with \( p \approx 0.9 \).

3 Results for planar shocks

3.1 Early time dynamics and non-universal transient effects

Let us begin by focusing on the energy density produced by Gaussian shock collisions. In Fig. 2 we plot the rescaled energy density \( \frac{T^{00}}{\mu^4} \), for Gaussian shock profiles with widths \( w = 5 w_0 \) (top) and \( w = w_0 \) (bottom). The shocks approach each other at the speed of light in the

For comparison, Ref \[15\] used \( w = 10 w_0 \) and Ref \[1\] used \( w \) in the range 0.66 \( w_0 \) till 25 \( w_0 \).
symmetric collisions

narrow shocks, $w = 0.075/\mu$
symmetric collisions: results

- no surviving remnants on lightcone
- no significant difference between wide & narrow shocks
  - hydrodynamic regime:
    - \( \mu t_{\text{hydro}} \approx 2 \)
  - negligible departure from boost invariant flow:
    \[
    u^\tau = 1, \quad u^\xi = u^\perp = 0.
    \]
- universal Gaussian rapidity dependence:

  \[
  \epsilon(\xi, \tau_{\text{init}}) = \mu^4 A(\mu w) e^{-\frac{1}{2} \xi^2 / \sigma(\mu w)^2}
  \]

- For \( \tau = 3.5/\mu \): \( A(\mu w) \approx 0.14 + 0.15 \mu w - 0.025(\mu w)^2 \)
  \[
  \sigma(\mu w) \approx 0.96 - 0.49 \mu w + 0.13(\mu w)^2
  \]

Chesler, Kilbertus, van der Schee
symmetric collisions: results

\[ w = w_o, 2w_o, \ldots, 6w_o \]

\[ w = 7w_o \]

\[ \xi / \xi_{\text{FWHM}} \]

\[ \epsilon / (\epsilon(\xi = 0)) \]

\[ \epsilon / (\epsilon(\xi = 0)) \]

\[ \xi / \xi_{\text{FWHM}} \]

Chesler, Kilbertus, van der Schee

L. Yaffe, Holographic QCD, July 2019
asymmetric planar shock collisions

S. Waeber, A. Rabenstein, A. Schäfer, L. Yaffe, 1906.05086
asymmetric planar collisions

- Incoming shocks: Gaussian energy density profiles, widths $w_\pm$:
  \[ h_\pm(z) \equiv \mu_\pm^3 (2\pi w_\pm^2)^{-1/2} e^{-\frac{1}{2} z^2 / w_\pm^2} \]
  - CM frame $\Rightarrow$ equal longitudinally integrated energy density, $\mu_+ = \mu_-$

- Single shock metric (Fefferman-Graham coordinates):
  \[ ds^2 = \tilde{\rho}^{-2} \left( -d\tilde{x}_+ d\tilde{x}_- + d\tilde{x}_\perp^2 + d\tilde{\rho}^2 \right) + \tilde{\rho}^2 h(\tilde{x}_\pm) d\tilde{x}_\mp^2 \quad \tilde{x}_\pm \equiv \tilde{t} \pm \tilde{z} \]

- Time evolution: superpose two counter-propagating shocks, transform to null infalling coordinates, use characteristic formulation of general relativity, spectral methods for PDEs

  - Details: horizon finding, nested linear radial ODEs, Chebyshev & Fourier grids, domain decomposition, filtering, RK4 time stepping, ...

L. Yaffe, Holographic QCD, July 2019
asymmetric collisions: results

- Energy density:

- Outgoing decaying “remnants” with deposited energy density in forward lightcone, just like symmetric collisions
asymmetric collisions: results

- Receding energy density maxima:

- Same power-law decay seen in symmetric case
asymmetric collisions: results

- Hydrodynamic domain (residual $\Delta < 0.15$):

\[ w_{\pm} = 0.075. \quad (w_+, w_-) = (0.1, 0.25) \]

- Weak sensitivity to widths, $t_{\text{hydro}} \approx 2/\mu$
asymmetric collisions: results

- Deviation from boost invariant flow \((u^\tau = 1, \ u^\xi = u^\perp = 0)\):

\[(w_+, w_-) = (0.075, 0.35) \text{ (black)} \]
\[w_\pm = 0.075 \text{ (red)} \]
\[w_\pm = 0.35 \text{ (green)} \]

- Negligible deviation
asymmetric collisions: results

- Proper energy density vs. rapidity: remains essentially Gaussian

- Near-identical to shifted geometric mean of symmetric collision results:

\[
\xi(\mu_+, \mu_-) \approx \Xi \frac{w_+ - w_-}{w_+ + w_-} \quad \Xi \approx 7 \times 10^{-2}
\]

\[
(w_+, w_-) = (0.075, 0.35) \quad \tau = 2
\]

\[
(w_+, w_-) = (0.075, 0.25)
\]
asymmetric collisions: results

- Gaussian rapidity distribution amplitude, asymmetric collisions vs. geometric mean of symmetric collisions:

  \[(w_+ , w_-) = (0.075, 0.35)\]

  \[(w_+ , w_-) = (0.1, 0.25)\]

- Near perfect agreement
asymmetric collisions: results

- Gaussian rapidity dependence of proper energy density:

\[ \epsilon(\xi, \tau_{\text{init}}) = \mu^4 A(\mu w_+, \mu w_-) e^{-\frac{1}{2}(\xi-\xi(\mu w_+, \mu w_-))^2/\sigma(\mu w_+, \mu w_-)^2} \]

- Rapidity shift: \( \bar{\xi}(\mu w_+, \mu w_-) \approx \Xi \frac{w_+ - w_-}{w_+ + w_-} \).

- Asymmetric collisions \( \approx \) geometric mean of symmetric collisions:

  - Amplitude: \( A(\mu w_+, \mu w_-) \approx \sqrt{A(\mu w_+) A(\mu w_-)} \)
  
  - Width: \( \sigma(\mu w_+, \mu w_-) \approx [\frac{1}{2}\sigma(\mu w_+)^{-2} + \frac{1}{2}\sigma(\mu w_-)^{-2}]^{-1/2} \)
colliding localized shocks ("nuclei")

P. Chesler and L. Yaffe, 1501.04644
P. Chesler, 1506.02209, 1601.01583
localized shock collisions

- Requires solving 5D PDEs
- Characteristic methods ill-suited to distributed computing
- Feasible (with difficulty) on modern multi-core desktops
  - Memory & time intensive: 10+ Gbyte, multiple months
  - Limited dynamic range: far from realistic aspect ratios
- But possible!
localized collisions
hydrodynamic residual

\[
\Delta \equiv \frac{1}{\bar{p}} \sqrt{\Delta T_{\mu\nu} \Delta T^{\mu\nu}}, \quad \Delta T^{\mu\nu} \equiv T^{\mu\nu} - T_{\text{hydro}}^{\mu\nu} \quad \bar{p} \equiv \frac{\epsilon}{3}
\]
flow velocity

$t = 4 \quad \text{non-hydro regions excised}$

substantial radial flow: 

$v_{\perp}(x_{\perp} = 5) \approx 0.3$

$v_{\parallel}^{\text{max}} \approx 0.64$
localized collisions: lessons

- “pre-hydro” development of transverse flow
- rapid equilibration, $t_{\text{hydro}} T_{\text{eff}} \approx 0.3$
- extreme hydrodynamics:
  - huge anisotropy but well-behaved gradient expansion
  - works down to $R T_{\text{eff}} \approx 0.5-1$
    - compatible with interpretations of high multiplicity $\rho-\rho$ collisions as producing deconfined quark-gluon plasma exhibiting collective flow
modeling pre-hydro evolution

- Real nuclear projectiles:
  - huge aspect ratio, $\gamma \geq 1000$
  - initial state fluctuations relevant
  - transverse gradients $\ll$ longitudinal gradients

⇒ Use planar shock results to model “pixel by pixel”:
modeling pre-hydro evolution

Given model of initial nuclear energy density, use planar shock results to predict pre-hydro evolution “pixel by pixel”:

- For each pixel $j$:
  - boost to CM frame of individual pixel
  - use planar shock results to predict $T^{\mu\nu}(j)$
  - transform back
- Combine all pixels

Hydro initial data with slow transverse variation

Similar proposal w/o width dependence:
W. van der Schee, B. Schenke, 1507.08195
next steps, open questions

- Explanation for Gaussian rapidity dependence?
- Explicit comparison of localized shock collisions with planar shock based model
- Higher terms in transverse gradient expansion?
- Correlations, non-local observables
- Confrontation with real data
- Influence of transverse variations in hydrodynamization time
- ...