Neutron star mergers
Outline

Block I: Astrophysics of neutron star mergers

a) basic considerations
   - neutron stars
   - binary neutron stars
   - dynamics up to merger
   - merger process

b) neutron star mergers from different perspectives:
   - gravitational wave astronomy
   - gamma-ray bursts
   - nucleosynthesis
   - electromagnetic transients

Block II: Modeling of neutron star mergers
Block I: Astrophysics of neutron star mergers

a) Basics considerations

A) Some historic milestones

● 1920: Ernest Rutherford predicts the neutron


● 1932: discovery of neutron by Chadwick

● 1934: Baade and Zwicky propose:

  a) massive stars end their life as a supernova
  b) such explosions produce cosmic rays and
  c) a collapsed star made of neutrons
1939: Tolman and Oppenheimer & Volkoff calculate the internal structure of a neutron star by

a) "macroscopic": generalizing the Newtonian hydrostatic equilibrium condition ("Tolman-Oppenheimer-Volkoff", TOV) equations

b) "microscopic": neutron star matter described by a cold (T=0), ideal Fermi gas of neutrons,

main conclusions TOV:

⇒ existence of a "maximum mass" beyond which there is no stable solution

⇒ for the neutron Fermi gas equation of state (EOS): $M_{\text{max}} \approx 0.8 \, M_\odot$
Maximum neutron star mass:
- if one increases the mass of the object, the density increases;
- object is stable if the “new pressure” can balance the “new density”;
  
- this means that \( \frac{dP}{d\rho} \) determines the stability
- this is the sound speed! 
- rigidity

- since the sound speed is limited to \( c \), there comes a point when the pressure no longer can balance gravity
  \( \Rightarrow \) collapse

- 1964 Hoyle, Narlikar and Wheeler predict neutron stars rotate rapidly
- 1965 Hewish and Okoye discover intense radio source in Crab nebula
- 1966 Wheeler predicts Crab nebula powered by rotating neutron star
- 1966 Colgate & White: supernova simulations that lead to neutron stars
- 1966 First pulsar model by Pacini

- 1967 Jocelyn Bell Burnell:
  “…pulses of 1.3 s period…”

- 1967 Bell, Pilkington, Scott & Collins discover “first” PSR 1919+21
- 1968 Crab pulsar discovered, found to slow down, connection with supernovae
- 1968 T. Gold: “pulsars= magnetized rotating neutron stars”
- 1974 Nobel Prize to A. Hewish (not to Bell or Okoye)
Who’re they? & why so happy?

(shamelessly stolen from Madappa Prakash 2006)
1974 R. Hulse and J. Taylor discover binary system made of neutron stars

1979 PSR 1919+21 as album cover for *Unknown Pleasures* by Joy Division

1982 First millisecond pulsar PSR B1937+21

1992 First exoplanet around pulsar PSR B1257+12

1993 Nobel Prize for Hulse and Taylor

1998 First “magnetar” SGR 1806-20 by Kouveliotou et al.

2004 “magnetar” giant flare from SGR 1806-20: largest burst of energy in Galaxy since 1604 ($\approx 2 \times 10^{46}$ erg)

2004 PSR J1748-2446ad: fastest known rotation rate, 716 Hz

2005 PSR J0737-3039: first binary with two pulsars

2009 PSR J1614-2230: $M_{\text{ns}} = 1.97 \pm 0.04 \, M_{\text{sol}}$
Some amazing facts about neutron stars

- densest observable objects in the Universe: $\sim 10^{15} \text{ g/cm}^3$

- fastest spinning object PSR J1748-2446ad: rotates with 716 Hz; if $R=15$ km this corresponds to $c/4$

- strongest magnetic fields: $\sim 10^{15}$ G (typical field in Sun 1 G)

- fastest measured massive object in Galaxy (PSR 1508+55): 1083 km/s

- highest temperature since Big Bang: $\sim 7 \times 10^{11}$ K at birth

- only place in the Universe (apart from Big Bang) where even neutrinos are trapped
B) Implications of some observed numbers

Order of magnitude estimate I: “rotation speed constrains average density”

- shortest period pulsar (“Ter5ad”; PSR J1748-2446ad): $P = 1.39$ ms, $\omega = 4500 \text{ s}^{-1}$

  ⇒ IF held together by gravity: “surface centr. accel. < gravitational accel.”

\[
\frac{m(\omega R)^2}{R} < \frac{GmM}{R^2}
\]

\[
\frac{3\omega^2}{4\pi G} < \bar{\rho} = \frac{M}{(4\pi/3)R^3}
\]

⇒ $\bar{\rho} > 8 \times 10^{13} \text{ gcm}^{-3}$ central density might be substantially higher

⇒ densest objects in the observable Universe
Order of magnitude estimate II: “rotation speed constrains radius”

⇒ Causality: “surface velocity must be smaller than c”

⇒ \( \omega R < c \)

\[
R < \frac{cP}{2\pi} \approx 65 \text{ km}
\]
C) Birth in a Supernova

Supernovae in the Universe

1–10 supernovae explode every second in the Universe

~2 per 100 years in the Milky Way

energy release:
- in radiation: \( \sim 10^{49} \) erg
- in kinetic energy ejecta: \( \sim 10^{51} \) erg
- in neutrinos (core-collapse SN): \( \sim 10^{53} \) erg

types:
- thermonuclear explosion of white dwarf ("type Ia"; energy source: nuclear binding energy)
- core-collapse (type II, type b/c, ...; energy source: gravitational binding energy)
- Neutron stars are formed in core-collapse supernovae of stars more massive than \( \approx 8 \) solar masses.

- Progenitor star has "onion-like" structure with iron-core.

- Collapse sets in when the iron core reaches its "Chandrasekhar-mass" (electrons become fully relativistic, adiab. exponent \( \Gamma \to 4/3 \) \( \Rightarrow \) gas pressure cannot react approp. on perturbation; at \( \approx 1.3 \, M_{\odot} \)).
Sketch of core-collapse mechanism
 Confirmation of the basic mechanism: **SN 1987A**

- mass $\approx 17 \, M_{\text{sol}}$
- $T_{\text{eff}} \approx 17,000 \, \text{K}$
- star has disappeared
- detected neutrinos confirm birth of neutron star
- to date no pulsar or neutron star detected
D) Neutrino emission from proto-neutron stars

Order of magnitude estimate III: “neutrinos in a new-born neutron star”

- **Concept** electron fraction $Y_e = \frac{\#\ electrons}{\#\ baryons}$

  examples:
  - $^4\text{He}$: $Y_e = \frac{2\ electrons}{4\ nucleons} = 0.5$
  - pure neutron matter: $Y_e = 0$
  - most stable nuclei: $Y_e \approx 0.5$

- **Source of energy:**
  grav. binding energy from collapse of iron core to proto-neutron star (PNS)

- **Used subsequent order of magnitude estimates:**
  - $R_{\text{PNS}} \approx 20\ km$
  - $Y_{e,\text{PNS}} \approx 0.2$
  - $M_{\text{PNS}} \approx 1.2\ M_{\odot}$

- **Nuclear equation of state (EOS) is very stiff**
  $\Rightarrow$ (nearly) same density everywhere

  $\Rightarrow E_{\text{grav}} \approx -\frac{3}{5} G M_{\text{PNS}}^2/R_{\text{PNS}} \approx 1 \times 10^{53}\ erg \ (M_{\text{PNS}}/1.2\ M_{\odot})^2 (20\ km/R_{\text{PNS}})$
● (roughly) how many neutrinos?
⇒ assume: “every proton transformed into neutron by electron capture (EC)
\[ e + p \rightarrow n + \nu_e \] produces neutrino”

# nucleons = \( M_{\text{PNS}}/m_u \approx 1.4 \times 10^{57} \), \( m_u = 1.66 \times 10^{-24} \) g nucleon mass

# protons \sim 0.5 \# nucleons \sim \# neutrinos \sim 7 \times 10^{56}

● \( \langle E \rangle_{\nu} \sim E_{\text{grav}}/(\#\text{neutrinos}) \sim 90 \text{ MeV} \) (very crude; the neutrinos that escape have lower energies; detailed calculations yield \sim 20 \text{ MeV})

● Is the PNS transparent to its neutrinos?
⇒ scattering cross section is

\[
\sigma \approx \frac{1}{4} \sigma_0 \left( \frac{E_\nu}{m_e c^2} \right)^2
\]

\text{comments: - quadratic in neutrino energy}
⇒ more opaque for higher energy neutrinos

- reference cross section \( \sigma_0 \) \text{ very small}: \( \sigma_0 = 1.76 \times 10^{-44} \text{ cm}^2 \)
**mean free path**

\[ \lambda = \frac{1}{n\sigma} \]

n nucleon number density

**concept optical depth:**

\[ \tau \equiv \int \frac{ds}{\lambda(r')} = \int \frac{n(r')\sigma(r')}{\sigma} \]

physical interpretation:
“measure of the number of interactions to escape”

**for PNS:**

\[ n \sim \rho/m_u \sim 4 \times 10^{37} \text{ cm}^{-3} \Rightarrow \]

\[ \lambda \approx 2m \left( \frac{90 \text{ MeV}}{E_\nu} \right)^2 \left( \frac{R_{\text{PNS}}}{20 \text{ km}} \right)^3 \left( \frac{1.2M_\odot}{M_{\text{PNS}}} \right) \ll R_{\text{PNS}} \]

⇒ “proto-neutron star is opaque to its own neutrinos”
neutrino’s “random walk” out of PNS:
   a) How many scatterings before escaping?
   b) How long does it take (“diffusion time”)?

- average distance between scatterings: $\lambda$
- average time between scatterings: $\Delta t = \lambda/c$
- displacement vector after $N$ scatterings: $\vec{R} = \sum_i \vec{r}_i$
- assume i) $\tau > 1$ and ii) scattering angle is random $\Rightarrow \langle \vec{R} \rangle = 0$
  $\Rightarrow$ average square
  $\langle \vec{R}^2 \rangle = \langle \vec{r}_1^2 \rangle + \langle \vec{r}_2^2 \rangle + \ldots + 2\langle \vec{r}_1 \cdot \vec{r}_2 \rangle + 2\langle \vec{r}_1 \cdot \vec{r}_3 \rangle + \ldots$
  i.e.
  $\langle \vec{R}^2 \rangle = N\lambda^2$
- define “rms net displacement”
  $l_* \equiv \sqrt{\langle \vec{R}^2 \rangle} = \sqrt{N}\lambda$
• "escape" means "rms net displacement = system size": \( l_* = R \)

• number scatterings to escape: \( N = \frac{R^2}{\lambda^2} \)

• total time to escape: \( t_{\text{tot}} = N \Delta t = \frac{R^2 \lambda}{\lambda^2 c} \)

• "diffusion time": \( t_{\text{diff}} \sim \frac{R^2}{\lambda c} \)
- **time to diffuse out** of PNS centre, i.e. a distance $R_{\text{PNS}}$ (“random walk”):

$$\tau_{\text{diff}} \sim \frac{R_{\text{PNS}}^2}{\lambda c} \sim 1 \text{ s}$$

- **typical neutrino luminosity**

$$L_\nu \sim \frac{E_{\text{grav}}}{\tau_{\text{diff}}} \sim 10^{53} \text{ erg/s}$$

- **detailed calculations** (e.g. Liebendörfer et al. 2001) yield:
  - $\langle E \rangle_\nu \approx 20 \text{ MeV}$
  - $L_\nu \approx 5 \times 10^{52} \text{ erg/s}$

- **transfer to neutron star mergers**:
  - in order of magnitude still correct
  - but domination by anti-neutrinos ($\text{from } e^+ + n \rightarrow p + \bar{\nu}_e$)
E) Neutron stars as giant nuclei

- **nuclear binding energy** $B$: 
  \[ M(A, Z) = N m_n + Z m_p - \frac{B(A, Z)}{c^2} \]

  - $M$: nuclear mass
  - $A$: nucleon number
  - $Z$: proton number
  - $N$: neutron number

- **binding energy per nucleon** $B/A$:

  $\Rightarrow$ for large nuclei: 
  $B/A \approx \text{constant}$

  $\Rightarrow$ binding energy proportional to nucleon number

  $B \propto A$
nucleon does NOT interact with all other nucleons

finite range interaction like in a liquid

“Liquid drop model of atomic nucleus” (Weizsäcker 1935):

- interaction only with direct neighbors
- nucleons at surface are weaker bound (surface tension effects)

strategy:
- determine different physical effects with the functional dependences
- constants from fit to nuclear data

volume nucleus \( \propto R^3 \propto A \Rightarrow \)

\[ R_{\text{nucleus}} \propto A^{1/3} \]

\[ R_{\text{nucleus}} = R_0 \ A^{1/3} \]

scatt. experiments: “nucleon radius”

\[ R_0 = 1.2 \text{ fm} \]
i.e. $B \propto A$ means “binding energy proportional to volume”

- “volume energy”
  \[ B_1 = a_{\text{vol}} A \]

- “nucleons as surface weaker bound”
  \[ B_2 = -a_{\text{surf}} A^{2/3} \]

- “symmetric nuclei (N=Z) particularly well bound”
  \[ B_3 = -a_{\text{sym}} A \delta^2 \]
  \[ \delta = 1 - 2Z/A = 1 - 2x_p \]

- “Coulomb repulsion reduces binding energy”
  \[ B_4 = -\frac{3e^2}{5R_0} Z^2 A^{-1/3} \]

- “pairing of nucleons”
  \[ B_5 = \Delta(A, Z) \]
  \[ \Delta(A,Z) = \begin{cases} \Delta_0 & \text{for even } Z\&N \ (A \text{ even}) \\ 0 & \text{for odd } A \\ -\Delta_0 & \text{for off } Z\&N \ (A \text{ even}) \end{cases} \]
parameters determined from more than 2000 nuclei (A>8)

⇒ binding energy as function of A and Z

"proton drip line",
\[ \frac{\partial B}{\partial Z} = 0 \]

"neutron drip line",
\[ \frac{\partial B}{\partial N} = 0 \]

⇒ What happens if we apply this concept to the neutron star as a hole?
(i.e. in the limit A → ∞; ⇒ Landau’s “giant nuclei”)

(source: Wikipedia)
• side remark: estimate of the density of close-packed nucleons

- assume “radius” $R_0 = 1.2$ fm

- “close-packed”: 1 nucleon per sphere of radius $R_0$

\[
\rho_0 \sim \frac{1.66 \times 10^{-24} \text{ g}}{(4\pi/3)R_0^3} \approx 2.3 \times 10^{14} \frac{\text{ g}}{\text{ cm}^3}
\]

- “nuclear saturation density”

\[
\bar{\rho}_{\text{sat}} = 2.5 \times 10^{14} \frac{\text{ g}}{\text{ cm}^3}
\]

- average density typical neutron star:

\[
\bar{\rho}_{\text{ns}} = \frac{1.4 M_\odot}{(4\pi/3)(12 \text{ km})^3} \approx 4 \times 10^{14} \frac{\text{ g}}{\text{ cm}^3}
\]
• binding energy:

\[ B = a_{\text{vol}} A - a_{\text{surf}} A^{2/3} - a_{\text{sym}} A(1 - 2x_p)^2 - a_{\text{coul}} Z^2 A^{-1/3} + \Delta(A, Z) \]

• energy per nucleon:

\[ \frac{B}{A} = a_{\text{vol}} - a_{\text{surf}} A^{-1/3} - a_{\text{sym}} (1 - 2x_p)^2 - a_{\text{coul}} Z^2 A^{-4/3} + \frac{\Delta(A, Z)}{A} \]

⇒ diverges unless \( x_p = 0 \)

⇒ focus on pure neutron matter: \( Z = 0 \) & \( x_p = 0 \)

⇒ in limit \( A \to \infty \):

\[ \frac{B}{A} = a_{\text{vol}} - a_{\text{sym}} = 10.6 \text{ MeV} \]

⇒ “pure neutron matter is unbound!!!”

⇒ So what about neutron stars?
they need gravity to be bound!

\[ E_{\text{grav}} \approx - \frac{3GM^*}{5R^*} = - \frac{3Gm_{\text{nuc}}^2}{5r_0} A^{2/3} \]

⇒ bound for a sufficiently large nucleon number,

there is a MINIMUM neutron star mass (~0.07 \( M_{\text{sol}} \))!!

of course, reality is much more complicated...

- GR
- protons/other particles are present
- nuclear matter is compressible
- ...

...
F) Neutron stars as relativistic objects

How to calculate the structure of a Newtonian star?

“hydrostatic equilibrium = pressure gradients balance gravity”

- net force due to pressure gradient in shell:
  \[ F_P = 4\pi r^2 \frac{dP}{dr} \]

- gravitational force due to mass of shell:
  \[ F_g = -\frac{Gm(r)dm}{r^2} = -\frac{4\pi r^2 Gm(r)\rho dr}{r^2} \]

\[ \Rightarrow \frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2} \]

\[ \frac{dm}{dr} = 4\pi r^2 \rho \]

pressure: “out”  gravity: “in”

in practice: - start from central value for \( \rho/P \)
  - integrate outward until \( P \leq 0 \) = stellar surface
What changes for a relativistic star?

- all forms of energy are sources of gravity (i.e. also the pressure!)
- gravity curves space-time

“Tolman-Oppenheimer-Volkoff equation”

\[
\frac{dP}{dr} = - \frac{G\epsilon(r)m(r)}{c^2 r^2} \times \left[ 1 + \frac{P(r)}{\epsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 P(r)}{m(r)c^2} \right] \left[ 1 - \frac{1}{2Gm(r)c^2} \right] > 1 \times > 1 \times > 1
\]

comments:

- all corrective terms are > 1, they enhance gravity

- pressure now appears on both sides of the equation, i.e. it acts “out” and “in”

- if mass is increased, the pressure is increased which acts as addition source of gravitational field
  ⇒ at a critical mass gravitational collapse sets in, i.e. a neutron star also has a maximum possible mass!

- if GR is the correct theory, this maximum mass depends only on the equation of state
What is the equation of state?

simplest (but unrealistic!): a Fermi gas of non-interacting neutrons

- assume pressure is provided by non-interacting Fermi-gas following a Fermi-Dirac distribution

\[
f(E) = \frac{1}{\exp\left(\frac{E-\mu}{k_B T}\right) + 1}
\]

- number, energy density and pressure can then be calculated according to the standard thermodynamical expressions

\[
n = \frac{g}{h^3} \int f(E) d^3 p
\]

number density

\[
\epsilon = \frac{g}{h^3} \int E f(E) d^3 p
\]

energy density

\[
P = \frac{1}{3} \frac{g}{h^3} \int p v f(E) d^3 p
\]

pressure
Stability condition

- **equilibrium solution, stable?**

- **pert. 1: - compression** ➞ equil. star at this \( \rho \) needs larger mass ➞ bounce back
- **pert. 2: - decompression** ➞ equil. star at this \( \rho \) has lower mass ➞ bounce back
- **pert. 3: - compression** ➞ equil. star at this \( \rho \) has smaller mass ➞ further \( \rho \) increase
- **pert. 4: - decompression** ➞ equil. star at this \( \rho \) has larger mass ➞ further \( \rho \) decrease

\[
\frac{\partial M_*(\rho_c)}{\partial \rho_c} > 0
\]

necessary condition for stability
this is the approach originally taken by Oppenheimer & Volkoff (1939)

On Massive Neutron Cores

J. R. Oppenheimer and G. M. Volkoff

Department of Physics, University of California, Berkeley, California

(Received January 3, 1939)

It has been suggested that, when the pressure within stellar matter becomes high enough, a new phase consisting of neutrons will be formed. In this paper we study the gravitational equilibrium of masses of neutrons, using the equation of state for a cold Fermi gas, and general relativity. For masses under $\frac{3}{4} \odot$ only one equilibrium solution exists, which is approximately described by the nonrelativistic Fermi equation of state and Newtonian gravitational theory. For masses $\frac{3}{4} \odot < m < \frac{3}{4} \odot$ two solutions exist, one stable and quasi-Newtonian, one more condensed, and unstable. For masses greater than $\frac{3}{4} \odot$ there are no static equilibrium solutions. These results are qualitatively confirmed by comparison with suitably chosen special cases of the analytic solutions recently discovered by Tolman. A discussion of the probable effect of deviations from the Fermi equation of state suggests that actual stellar matter after the exhaustion of thermonuclear sources of energy will, if massive enough, contract indefinitely, although more and more slowly, never reaching true equilibrium.

$\Rightarrow$ upper mass limit $\sim 0.8$ solar masses!

• major shortcoming: no strong interaction included!
Repetition: Some basic thermodynamic relations

• Thermodynamic potentials

• generic form: \( \Xi = \Xi(X_1, X_2, ..., X_n) \) depending on \( X_1, X_2, ..., X_n \)

• examples (k particle species in volume V):
  - \( S = S(E, V, N_1, ..., N_k) \) entropy
  - \( E = E(S, V, N_1, ..., N_k) \) internal energy

• variation of potentials \( \Rightarrow \) differentials

\[
d\Xi = \xi_1 \, dX_1 + \xi_2 \, d\Xi_2 + ... \xi_n \, d\Xi_n
\]

\( \Rightarrow \) pairs of conjugate variables ("extensive \( \leftrightarrow \) intensive") \( X_l \leftrightarrow \xi_l \)
example: internal energy
\[ dE = TdS - PdV + \mu_1 N_1 + \ldots + \mu_k N_k \]

⇒ temperature
\[ T = \frac{\partial E}{\partial S} \bigg|_{V,N_i} = T(S, V, N_i) \]

pressure
\[ P = -\frac{\partial E}{\partial V} \bigg|_{S,N_i} = P(S, V, N_i) \]

chemical potential
\[ \mu_i = \frac{\partial E}{\partial N_i} \bigg|_{S,V,N_j \neq i} = \mu_i(S, V, N_k) \]
Which **matter constituents** are present (in equilibrium)?

- only particles that cannot decay!

- what about the neutron?

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

decays until no more final states (electrons) are available, i.e. until “available energy” \( (m_n - m_p)c^2 < \) “energy required for new electron on top of Fermi sea”

“Final state blocking” or “Pauli blocking”

- **Fermi-energy** (no int.)

\[ E_F = \sqrt{p_F c^2 + mc^2} \]

- **Fermi-momentum** \( p_F \)

no interaction, ultra-rel.

\[ p_F = \frac{\hbar}{3}\pi^2 n \]^{1/3}

- relativistic **chemical potential** = Fermi energy
• decay of neutron will continue until an equilibrium is reached, characterized by

\[ \mu_n = \mu_p + \mu_e + \mu_\nu (= 0) \]

“β-equilibrium”

• at \( \rho = 8.7 \times 10^{12} \text{ g cm}^{-3} \): electron chemical potential \( \mu_e = 105 \text{ MeV} = m_\mu c^2 \), i.e. myon starts to appear \( e^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_e \)

• tauon would appear at \( \rho = 8.2 \times 10^{16} \text{ g cm}^{-3} \): no need to be considered

• electrons and myons are the only leptonic components (initially neutrinos)

\[ \Rightarrow \text{Also transformation of nucleons into heavier baryons?} \]
- Baryons of lowest mass are made of u, d, s quarks:
  - **baryon octet:** n, p, \(\Lambda\), \(\Sigma^{-/0/+}\), \(\Xi^{-/0/+}\)

- Nucleons: "hyperons" (= baryons with strangeness content)

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**Quantum numbers**
- Baryon number \(B = 1\)
- Strangeness \(S\)
- Isospin component \(T_3\)
- Charge number \(Q\)
- Spin \(I = 1/2\)
- Parity \(P = +\)

- Quark content and masses:
  - \(p\) uud \(938.272 \text{ MeV}/c^2\)
  - \(n\) udd \(939.565 \text{ MeV}/c^2\)
  - \(\Lambda\) uds \(1115.68 \text{ MeV}/c^2\)
  - \(\Sigma^+\) uus \(1189.37 \text{ MeV}/c^2\)
  - \(\Sigma^0\) uds \(1192.64 \text{ MeV}/c^2\)
  - \(\Sigma^-\) dds \(1197.45 \text{ MeV}/c^2\)
  - \(\Xi^0\) uss \(1314.86 \text{ MeV}/c^2\)
  - \(\Xi^-\) dss \(1321.71 \text{ MeV}/c^2\)
How to go beyond the non-interacting Fermi-gas?

- many different approaches exist, all are technically rather involved

- here: restriction to very (!) basic ideas

- most commonly used: a) non-relativistic potential models
  b) relativistic, field theoretical models ("Walecka-type models")
Nuclear matter properties near saturation density

- **Taylor expansion of energy density** in 3 variables ($x = n_p/(n_p+n_n)$):

\[
e(n, T, x) = n \left[ B + \frac{K}{18} \left( 1 - \frac{n}{n_0} \right)^2 + S_v \frac{n}{n_0} (1 - 2x)^2 + a \left( \frac{n_0}{n} \right)^{2/3} T^2 \right]
\]

- $n$: nucleon number density
- $n_0$: nuclear saturation density, $n_0 = 0.16$ fm\(^{-3}\)
- $B$: binding energy of cold, symmetric ($n_n=n_p$, i.e. $x=1/2$) nuclear matter, $B \approx -16$ MeV
- $K$: compressibility, $K \approx 225$ MeV
- $S_v$: nuclear symmetry energy, $S_v \approx 30$ MeV
- $a$: nuclear level density parameter, $a \approx (15 \text{ MeV})^{-1}$

- parameters determined by experiments
- expansion becomes questionable too far from saturation...
equation of state from standard thermodynamic relations

e.g. pressure  \[ P = -\frac{\partial E}{\partial V} = n^2 \frac{\partial (\epsilon/n)}{\partial n} \]

comments:

- no mechanism build in to prevent sound speed, \( c_s = (\partial P/\partial \rho)_s^{1/2} \) from becoming larger than \( c \) ("acausal")

- for low/moderate mass neutron stars that does not need to be a problem, but it may be near \( M_{\text{max}} \)
Relativistic field theoretical approach ("Walecka-type" EOS)

Why?
- Lorentz covariance desirable property
- spin-effects automatically included
- causality ($c_s < 1$) built-in by construction

How?
- start from Lagrangian density $\mathcal{L} = \mathcal{L}(L, B, M)$
  - leptons $L = \{e, \mu\}$
  - baryons $B = \{n, p, \Lambda, \sigma^+, \sigma^-, \sigma^0, \Xi^0, \Xi^-\}$
  - mesons (mediate str. interac.) $M = \{\sigma, \omega_\mu\}$
- apply techniques from relativistic field theory (Euler-Lagrange equations, energy-momentum tensor $\Rightarrow$ pressure etc.) to obtain thermodynamic properties
physically perfectly plausible

uncertain at which densities “exotica” appear/how good parameters are for \( n > 2n_0 \)
• mass radius relationship for different EOSs:

PSR 1957+20
"black widdow"

PSR J1614-2230

PSR 1913+16

• for many EOS: radius \(\approx\) independent of mass

• M-R-relationship from Lane-Emden solution (Newt., \(P = K \, \rho^{\Gamma}\)): 

\[ R \propto M^{\frac{\Gamma - 2}{3\Gamma - 4}} \]

\( \Rightarrow \) in bulk \( \Gamma \approx 2 \)
• observed neutron star masses (from Lattimer 2012)

"Black widow pulsar" ~2.4 M$_{\text{sol}}$

Hulse-Taylor pulsar 1.44 M$_{\text{sol}}$

Suggests:
- nsns binaries may need very specific circumstances to form
- ns in nswd binaries may have accreted substantial amounts
How large can the maximum mass maximally be?

- around saturation density, $\rho_0 \approx 2.5 \times 10^{14}$ g cm$^{-3}$, essentially all EOSs agree with experiments

- use accepted EOS at lower densities (BDS = “Beaudet, Petrosian, Salpeter”), match it at a “matching density” with stiffest possible one where $c_s = 1$, $P = \rho c^2 - P_0$

- if matched at $\rho = 2\rho_0$ (Roads & Ruffini 1974)

\[ \Rightarrow \text{maximum possible } M_{\text{max}} \approx 3.2 \, M_{\text{sol}}! \]
G) Overview internal structure of neutron stars

- crust: iron-like nuclei
- “bulk”: likely n, p, e, μ
- as densities/chemical potentials of neutrons and protons increase further particles (Λ, Ξ ...) can plausibly appear

structure of a neutron star:

- masses: ≈ 1.4 $M_{\text{sol}}$
- radii: 10 – 15 km
- composition:
  - crust: “iron-like” nuclei, $Y_e \approx 0.46$, < 1 % of mass
  - bulk: neutrons, protons, electrons, muons, $Y_e \approx 0.1$
  - “core”: possibly “exotic” (hyperons, quark matter etc.)
I.2 Neutron star binaries form?

- Punchline: it takes rather special circumstances to form a neutron star binary/survive two supernova explosions etc.

- ONE possibility

(Stairs 2004)
system parameters
(Weisberg et al. 2010):

• orbital period: \( P_o = 7.75 \text{ h } (v \sim 10^{-3} \text{ c}) \)

• pulsar period: \( P_s = 59 \text{ ms} \)

• eccentricity: \( e = 0.62 \)

• periastron advance: \( (\delta \Phi)_{\text{PSR}} = 4^\circ \text{ yr}^{-1} \)
  \( \gg (\delta \Phi)_{\text{Mercury}} = 0.43'' \text{ yr}^{-1} \)

• masses:
  \( m_1 = 1.4398 \pm 0.0002 \)
  \( m_2 = 1.3886 \pm 0.0002 \)
• 5 “certain” systems (both masses accurately known)

• additionally: 5 “likely” systems (mass function consistent with ns) 
  (see Lorimer, Living Reviews in Relativity (2008))
  i.e. \( \approx 10 \) nsns binary systems

• no nsbh system has been observed so far

• expected merger rates:
  
  - nsns mergers: few times \( 10^{-5} \) yr\(^{-1} \) galaxy\(^{-1} \), large uncertainties
  
  - nsbh mergers: essentially unknown
    estimates from 0.01 x nsns-rate to 10 x nsns-rate
I.3 System dynamics up to merger

- for astrophysical questions it is crucial how the inspiral takes:
  - expected LIGO/VIRGO detection rate
  - IF nsns mergers produce gamma-ray bursts: “Starting from which redshift can one expect to see a nsns-merger?”
  - IF they are a major source of r-process: “Can the enrich already very early generations of stars?”

- for essentially all of the inspiral phase can the binary be treated to good approximation as a Newtonian point masses with small dissipative terms for energy and angular momentum loss

- assume:
  - spacetime is essentially flat
  - GW emission is dominated by the leading quadrupole terms (“quadrupole approximation”)
  - stars are point masses
loss of energy and ang. momentum... (Peters & Mathews 1963; Maggiore 2008)

\[
\frac{dE}{dt} = -\frac{32}{5} \frac{G^4 \mu^2 M^3}{c^5 a^5} \frac{1}{(1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right)
\]

\[
\frac{dL}{dt} = -\frac{32}{5} \frac{G^{7/2} \mu^2 M^{5/2}}{c^5 a^{7/2}} \frac{1}{(1 - e^2)^2} \left(1 + \frac{7}{8} e^2\right)
\]

...drives decrease in separation and eccentricity

\[
\frac{da}{dt} = -\frac{64}{5} \frac{G^3 \mu M^2}{c^5 a^3} \frac{1}{(1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right)
\]

\[
\frac{de}{dt} = -\frac{304}{15} \frac{G^3 \mu M^2}{c^5 a^4} \frac{e}{(1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2\right)
\]

⇒ sensitive functions of both semi-major axis a and eccentricity e!
• **most binary systems will have radiated away their eccentricity by the time of merger**, e.g. PSR 1913+16 $e_{\text{merger}} \approx 6 \times 10^{-6}$

• **may be different dynamically formed/perturbed binary systems (globular clusters)**

• **GW frequency evolution (circ. orbit)**

$$f_{\text{GW}} \approx 134 \text{ Hz} \left( \frac{1.21 M_\odot}{M_{\text{chirp}}} \right)^{5/8} \left( \frac{1 \text{s}}{\tau} \right)^{3/8}$$

```
"chirp mass": $M_{\text{chirp}} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$
```

time to coalescence

$\Rightarrow$ for about **17 minutes** sweeping through LIGO frequency range!
— a fraction of systems could merge essentially directly after birth
I.4 Merger dynamics and remnant properties

**Simulation ingredients:**

- 3D, Lagrangian Hydrodynamics (SPH) & (Newtonian) Gravity

- *equation of state:* density, temperature and composition dependent RMF equation of state (Shen et al. 1998)

- *neutrino emission:* opacity-dependent multi-flavour leakage scheme

**References:**

“standard case”
- masses close to 1.4 $M_{\text{sol}}$, slight asymmetry (1.3 and 1.4 $M_{\text{sol}}$)
- zero initial spins
- stars in cold $\beta$-equilibrium
- simulated time $\sim 15$ ms
- visualized: $\log(T \ [\text{MeV}])$ at given opt. depth
dito, but now upper half (z>0) of stars “chopped off”

• neutrino emission:

\[ L \approx 10^{53} \text{ erg/s} \]

\[ E_{\nu e} \approx 8 \text{ MeV} \]
\[ E_{\nu x} \approx 14 \text{ MeV} \]
\[ E_{\nu x} \approx 26 \text{ MeV} \]
• disk formation I (mass + velocity):
disk formation II (electron fraction $Y_e$ + velocity):
Dependence on mass ratio:

\[ [1.0 \, M_{\text{Sol}}, \, 2.0 \, M_{\text{Sol}}] \times [1.0 \, M_{\text{Sol}}, \, 2.0 \, M_{\text{Sol}}] \]

Asymmetry in masses leads to:

- pronounced single tidal tail
- larger ejected masses
- larger ejecta velocities
- larger el. mag. luminos. ("macronovae", radio flares)
variation on the theme: dynamical collisions of compact objects

- common wisdom: “...stars do not collide...”
- BUT:
  i) collision rate
  \[ \propto (\text{number density})^2 \]
  ii) cross-section enhanced by “gravitational focusing”

- surface escape velocity, \( \sim 0.3 \, c \)
- velocity dispersion, \( \sim 10 \, \text{km/s} \)

\[ \sigma \approx \frac{\sim 10^8 \, \text{for a ns}}{\text{in a globular cluster}} \]
• **collision rates** are difficult to estimate, large uncertainties, maybe a reasonable fraction of the nsns-merger rate (O’Leary et al. 2009, Lee et al. 2010,...)

• characterize **impact strength** of parabolic orbit:

\[ \beta = \frac{R_1 + R_2}{r_{\text{per}}} \]
Examples of collisions

**example 1:** $m_1=1.3$, $m_2=1.4$, $\beta=1$

(2D-cut through 3D simulation)
\[ m_1 = 1.3, \quad m_2 = 1.4, \quad \beta = 1 \]

(temperature at large optical depth)

no compression (\( \rho(t) < \rho_0 \))

peak temperature \( \approx 25 \text{ MeV} \)
example 2: a more central collision $\beta=2$
Example 3: neutron black hole collision $m_{ns}= 1.3 \, M_{\odot}$, $m_{bh} = 5 \, M_{\odot}$, $\beta=1$

“grazing impact”

$E_{\bar{v}_e} \approx 7 \, \text{MeV}$  
$E_{v_e} \approx 8 \, \text{MeV}$  
$E_{\nu_x} \approx 14 \, \text{MeV}$  

$\nu$-luminosity

$\text{tot. luminosity: } 6 \times 10^{52} \, \text{erg/s}$

$(t=120 \, \text{ms})$
Remnant properties

- central object: “Collapse to a black hole ???”

Yes, likely, but...

⇒ production of differentially rotating, “hyper-massive neutron star”, \( M \approx 2.5 \, M_{\text{sol}} \)

⇒ differential rotation is VERY efficient in stabilizing stars!

(4 \( M_{\text{sol}} \) white dwarfs \( \gg \) \( M_{\text{Chandrasekhar}} \approx 1.4 \, M_{\text{sol}} \); Ostriker & Bodenheimer 1967)
- **binary mass** $M_{DNS} > M_{th}$ : direct collapse to BH
  $M_{DNS} < M_{th}$ : collapse via “hyper-massive neutron star”

$$M_{th} \approx 1.35 M_{\text{max, TOV}} \geq 2.66 M_{\odot}$$

$\Rightarrow$ delayed collapse likely, (maybe both realized in nature)

$\Rightarrow$ can a stable neutron star remnant be safely ruled out?
Compact binary mergers: multi-physics events

-15 min  -1 min  t= 0  10 ms  100 ms  1s  1 day  1 year

- tidal deformation ns
- gravitational wave emission
- merger
- disk formation
- v-driven wind
- “fallback”
- black hole formation
- disk dissolution
- opt. transients
- radioact. ejecta
- radio flares
Relevance of compact binary mergers

a) **Fundamental physics**
   - Tests of theory of gravity
   - **Direct** detection of gravitational waves (LIGO, VIRGO, GEO,...; in advanced stages: detection out to \( z \sim 0.1 \))
   - Maximum neutron star mass: hadronic interaction at high density \((\rho \gg \rho_{\text{nuc}} \approx 2 \times 10^{14} \text{ g/cm}^3)\)

b) **Astrophysics**
   - Nucleosynthesis: are compact binary mergers sources of rapid neutron capture ("r-process") nuclei?
   - Gamma-ray bursts: do they power (about 1/3 of) the el.mag. most luminous explosions in the Universe?
I.5 Neutron star mergers from different perspectives

α) ...as gravitational wave sources

β) ...as Gamma-ray Burst central engines

γ) ...as producers of heavy elements

δ) ...as sources of electromagnetic transients:
   - “Macronovae”
   - radio flares
α) Binary neutron stars as gravitational wave sources

- LIGO & VIRGO detectors currently upgraded, increase sensitivity by factor > 10
  ⇒ accessible volume enhanced by > factor 1000

- expected to be online ~2015

Which additional signatures are produced by compact object encounters?

- first detections may be ambiguous/near detection threshold, additional signatures may give confidence, enhancement of detection efficiency

- “multi-messenger” approach will provide additional information on:
  - astrophysical events and their rates
  - their environment (host galaxies, ambient medium, ...)
  - the physics of the sources
β) Binary neutron stars as engines of gamma-ray bursts (GRBs)

Properties in a nutshell

- short, energetic flashes of gamma-rays
- accidental discovery by American spy satellites in 1967
- GRB distribution and some highlights from SWIFT mission
• **observed rate:** few per day (BATSE experiment)

• **duration:**
  
  (Gehrels et al. 2009)

  measured duration $T_{90}$

  corrected to source frame duration $T_{90}/(1+z)$

  ⇒ two categories:

  a) “long bursts”, $\sim 20$ s (source frame)
  
  b) “short bursts”, $\sim 0.2$ s

• **variability:** substantial variation on very short time scales, $\delta t \approx 1$ ms

• **spectra:**
  
  • “non-thermal”, i.e. optically thin, $\tau < 1$
  
  • short bursts have harder spectra

  \{ requires ultra-relativistic outflow ($\Gamma \approx 300$) \}
- host galaxies
  - long GRBs
    - star forming
    - bursts occur in brightest SF regions
    - nearby GRBs have SN Ib/c in coincidence
  - short GRBs
    - all kind of host galaxies, including ellipticals
    - bursts sometimes offset from host by few kpc
    - NO supernova in coincid. to deep limits

- redshift distribution

- burst energy: \[ E_{\text{GRB}} \approx 10^{53} \text{ erg } \left( \frac{\Omega}{4\pi} \right) \] (less for short GRBs)

- most popular progenitors
  - collapse massive star ("collapsar") for long GRBs
  - compact binary (ns + {ns, bh}) merger for short GRBs

- in some cases "late engine emission" \( \sim 100 - 1000s \)
"Compactness problem" (Schmidt 1978)

- see Felix Ryde’s lecture yesterday

- problem:
  - large number of $\sim$MeV photons
  - variation on $\delta t \sim 1$ ms
  - non-thermal spectrum

- naive estimate:

  $$\text{switching off the source occurs for observer on } \delta t > R/c$$
  $$\Rightarrow \text{"source must be small": } R < c \delta t \approx 300 \text{ km}$$

  $$\text{optical depth } \int \sigma n f_e \, ds \sim \sigma (N_\gamma / R^3) f_e R$$

  $$\tau \sim \frac{\sigma_T N_\gamma f_e}{R^2} \sim 10^{14}$$

  $$\Rightarrow \text{HUGE!}$$

- How can this produce non-thermal photons?
How does relativistic bulk motion help?

- **Photon energy is lower** in outflow restframe:
  
  \[ E'_{\gamma} = \frac{E}{\Gamma} \]

  \[ \Rightarrow \text{fraction of photons above pair-production threshold lower!} \]

- **Relativistic beaming**: source is not that small

  Only part of the source is observable!

  \[ \Theta \approx \frac{1}{\Gamma} \]

  \[ R < 2\Gamma^2 c \delta t \]

  \[ \Rightarrow \text{to avoid pair-production one needs } \Gamma \approx 100 \]
“Baryonic pollution problem”

- observed radiation is produced in ultra-relativistic outflows ($\Gamma \approx 100$), i.e. $v \approx 0.9999 \, c$

- sphere with (thermal) energy $E$ and baryonic mass expands to an asymptotic Lorentz factor
  \[ \Gamma_{\text{asy}} \approx \frac{E}{mc^2} \]

- to reach a Lorentz factor $\Gamma_{\text{asy}}$, it cannot be “loaded” with more mass than
  \[ m_{\text{crit}} = 2 \times 10^{-6} M_\odot \frac{E/10^{51} \text{erg}}{\Gamma_{\text{asy}}/300} \]

How does Nature separate mass from energy?
"standard engine": bh + accretion disk

FORMATION OF A GAMMA-RAY BURST could begin either with the merger of two neutron stars or with the collapse of a massive star. Both these events create a black hole with a disk of material around it. The hole-disk system, in turn, pumps out a jet of material at close to the speed of light. Shock waves within this material give off radiation.

“engine” ~ $10^7$ cm  
“GRB” ~ $10^{13}$ cm  
“afterglow” ~ $10^{18}$ cm
open issue: how is the relativistic outflow launched?

suggestions:

- **annihilation of neutrino-antineutrino pairs:** $\nu_i + \bar{\nu}_i \rightarrow e + e^+$
  
  $\Rightarrow$ see talk by Ivan Zalamea

- **various magnetic processes:**
  - spinning bh + magn. disk (Blandford & Znajek 1977)
  - “differentially rotating central object”: field amplification buoyant magnetic “bubbles”
    (Kluzniak & Ruderman 1998, ...)
  - ...

(from SR et al. 2003)

(from SR et al. 2013)
“Baryonic pollution problem”

L$_{\nu} \sim 10^{53}$ erg/s

Luminosities:
- $\sim 20$ MeV
- $\sim 4$ MeV

temperatures: $\sim 4$ MeV

grav. binding energy $E_{\text{grav}} \sim 30$ MeV/bar.

“baryon-free”: can ultra-relativistic outflow be launched here???

(1 MeV = $10^{10}$ K)
Neutrino-driven winds

- effects of neutrino-heating not accounted for in current SPH-code(s)
- approach:
  i) 3D merger simulation (MAGMA-code; SR&Price (2007))
  ii) mapping on 2D grid
  iii) use 2D neutrino-radiation-hydrodynamics calculation with VULCAN code (Livne et al. ‘04, Burrows et al. ‘07)

mass loss:

\[ \nu_e + n \rightarrow e + p \]
\[ \bar{\nu}_e + p \rightarrow e^+ + n \]

\[ \frac{dM}{dt} \sim 10^{-3} \frac{M_\odot}{s} \]

strong, non-relativistic (≈ 0.1c) baryonic outflow, no relativistic outflow possible as long as the central neutron star is alive!

relativistic outflow only after collapse to bh?
neutrino loss and gain at $t= 60$ ms:

major “gain regions”:
- outer ns-crust
- funnel region

MGFLD: Multi-group flux-limited diffusion
$S_n$: short-characteristic method
progenitor

“central engine”

ultra-rel. outflow

dissipation of kinetic energy

- compact
- active 0.1 - \(\sim\) 1s
- sometimes minutes - hours
- \(L_{\text{iso}} \sim 10^{-3} \, M_{\text{sol}} \, c^2 / s\)
- \(E_{\text{iso}} \sim 10^{-4} \, M_{\text{sol}} \, c^2\)

\(\Gamma \approx 100\)

\(\gamma\)-rays “afterglow”

- black hole + accretion disk?

- comment:
  these are VERY long time scales for a compact binary system!!

\[
\tau_{\text{dyn,ns}} = \frac{1}{\sqrt{G \bar{\rho}}} = 0.4 \, \text{ms} \left( \frac{10^{14} \, \text{g cm}^{-3}}{\bar{\rho}} \right)^{1/2}
\]

\[
\tau_{\text{dyn,bh}} \approx \frac{2\pi}{\omega_{K,\text{ISCO}}} \approx 1 \, \text{ms} \left( \frac{M_{\text{BH}}}{3 \, M_{\odot}} \right)
\]
Compact source of relativistic jets

- active for 0.1 - few seconds
- variation on ms time scale
- can launch jets with $\Gamma > 100$
- isotropic equivalent $\gamma$-energy $\sim 10^{51}$ erg/s
- isotropic equivalent total energy $\sim 10^{50}$ erg
- occurs in galaxies with + without star formation
- not related to a supernova
- is often offset from its host galaxy by few kpc
- can produce long-lived engine activities $\sim 1000$ s in some cases

nsns/nsbh mergers?
nsns / nsbh mergers are best model for short GRBs

(Eichler et al. 1989, Narayan et al. 1991, ...)

plausible alternatives?

**interesting suggestion:**

(MacFadyen et al. 2005)

- ns in X-ray binary system
- accretion until $M_{\text{max}}$ reached
- collapse $\Rightarrow$ bh $\Rightarrow$ jet
  $\Rightarrow$ interaction with companion
  $\Rightarrow$ late X-ray flares

**open issue:**

- is a disk left?
- or inside ISCO?
- does this depend on the neutron star structure/EOS?
γ) Binary neutron stars as production sites for heavy elements

- **Big Bang**: elements up to $^7\text{Li}/^7\text{Be}$
- **hydrostatic stellar burning**: up to “iron-group”
- **beyond “iron group”**: mainly neutron capture processes

⇒ essentially two neutron capture processes in nature:

- **rapid n-capture** ("r-process")
- **slow n-capture** ("s-process")
Basic reactions:

a) $n$-capture: $n + (Z,A) \rightarrow (Z,A+1)$

b) $\beta$-decay: $(Z,A) \rightarrow (Z+1,A) + e + \bar{\nu}_e$

*s- and r-process: basic mechanism*

- Far from stability
- "Rapid": $T_{n\text{-cap}} \ll T_\beta$
- Blocked at magic numbers
- Subsequent decay towards $\beta$-stability
r-process: what is the production site?

about half of the elements heavier than iron form via “r-process”

clues from metal-poor stars:
(at least) two sources:
  a) “weak” (Z<56): varying abundance patterns
  b) “strong” (Z>56): extremely robust abundance patterns

required physical conditions:
  a) high temperatures ~ 10⁹ K
  b) “lots of neutrons”, low \( Y_e \)
  c) short time scales

“standard” scenario, a core-collapse supernova, is seriously challenged producing all the “r-process” material
(e.g. Roberts et al. 2010, Fischer et al. 2010, Arcones & Janka 2011)

interesting alternative: decompression of neutron star matter, e.g. in a neutron star merger (Lattimer & Schramm 1974, Eichler et al. 1989, Freiburghaus et al. 1999, Roberts et al. 2011...)
How can a merger eject mass?

- **neutrino-driven winds**
  - ultra-relativistic outflow, $\Gamma > 100$
  - interaction region jet-wind, $\Gamma \sim$ few (?)
  - neutrinodriven winds $\langle v \rangle \approx 0.1c$

$\Rightarrow$ dynamic ejecta $\langle v \rangle \approx 0.1c$

Fig. with E. Gafton
Dynamical mass ejection

typical merger case: 1.3 & 1.4 $M_{\text{sol}}$, no spin

visualized: Ye value at given optical depth

total amount: 0.014 $M_{\text{sol}}$

extremely neutron rich: $Y_e \approx 0.03$, with small crust contaminations

velocity $v \approx 0.1$ c
"r-process in action" from Korobkin et al. 2012

$t : 0.00 \times 10^0 \ s / \ T : 10.96 \ \text{GK} / \ \rho_b : 8.71 \times 10^{12} \ \text{g/cm}^3$
all 23 cases produce practically identical abundance patterns; independent of the properties of the merging compact binary system

⇒ excellent candidate for source of heavy (A > 130), “robust” r-process component!!

(Even IF EOS- and/or GR-effects change the ejecta masses by factors of a few)

galactic r-process production rate
(Qian 2000)

galactic r-process production rate (95%, Kalogera et al. 2004)
Electromagnetic signals from ejecta I: Macronovae

- Radioactive decays in ejecta power optical/UV transients
- “Supernova-like”, but evolve faster and are dimmer
- From simulated ejecta properties:
  - “Standard” nsns merger: peak after $\approx 0.4$ days with $L_{\text{peak}} \approx 2 \times 10^{42}$ erg/s

$\Rightarrow$ But: opacities not well known!
Electromagnetic signals from ejecta II: Radio flares
(Nakar & Piran, 2011
Rosswog, Piran, Nakar, 2013;
Piran, Nakar, Rosswog, 2013)

- ejecta carry a lot of kinetic energy:

- interaction with ambient medium produces long-lived radio flares

ejecta properties from simulations + ($\varepsilon_e = \varepsilon_B = 0.1$; electron powerlaw dist. $p = 2.5$; merger at $10^{27} \text{ cm} \approx \text{detection horizon adv. LIGO/Virgo; synchrotron}$)

- peak after months - years

- “standard case” (2 x $1.4 \text{ M}_{\odot}$):
  - peak after $\sim 1$ year
  - with 0.04 mJy at 1.4 GHz
  - 0.2 mJy at 150 MHz

- sensitive to ambient matter density

mergers
\begin{align*}
\text{nsns: } & 2 \times 10^{50} \text{ erg} \ (1 \ldots 9 \times 10^{50} \text{ erg}) \\
\text{nsbh: } & \approx 10^{51} \text{ erg}
\end{align*}

collisions
\begin{align*}
\text{nsns: } & 1 \ldots 4 \times 10^{51} \text{ erg} \\
\text{nsbh: } & 6 \ldots 11 \times 10^{51} \text{ erg}
\end{align*}
Block II: Modeling of neutron star mergers

α) Equation of state

β) Neutrino emission

γ) Nuclear reactions

δ) Hydrodynamics
Table 1  Neutron star mass measurements, 1-σ uncertainties, $M > 0.9 \, M_\odot$ assumed

<table>
<thead>
<tr>
<th>Object</th>
<th>Mass ($M_\odot$)</th>
<th>Reference</th>
<th>Object</th>
<th>Mass ($M_\odot$)</th>
<th>Reference</th>
</tr>
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<tbody>
<tr>
<td>X-ray/optical binaries (mean, 1.568 $M_\odot$; error-weighted mean, 1.368 $M_\odot$)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4U 1700-377</td>
<td>2.44 +0.27</td>
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<td>Vela X-1</td>
<td>1.770 +0.083</td>
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<td>4U 1538-52</td>
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<tr>
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<td>LMC X-4</td>
<td>1.285 +0.051</td>
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<tr>
<td>Cen X-3</td>
<td>1.486 +0.082</td>
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<td>Her X-1</td>
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<td>J. Tomsick, private communication</td>
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<tr>
<td>Neutron star-neutron star binaries (mean, 1.322 $M_\odot$; error-weighted mean, 1.402 $M_\odot$)</td>
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<td>J1829+2456h</td>
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<td>42</td>
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<td>1.34 +0.13</td>
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</tr>
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<td>Companion</td>
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<td>J1906+0746</td>
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<td>Companion</td>
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<td>48</td>
<td>J0737-3039B</td>
<td>2.1249 ±0.0007</td>
<td>48</td>
</tr>
<tr>
<td>J1756-2251</td>
<td>1.318 ±0.017</td>
<td>49</td>
<td>Companion</td>
<td>1.258 ±0.017</td>
<td>49</td>
</tr>
<tr>
<td>J1807-2500B</td>
<td>1.3655 ±0.020</td>
<td>50</td>
<td>Companion?</td>
<td>1.2064 ±0.017</td>
<td>50</td>
</tr>
<tr>
<td>Neutron star-white dwarf binaries (mean, 1.543 $M_\odot$; error-weighted mean, 1.369 $M_\odot$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B2303+46</td>
<td>1.38 +0.06</td>
<td>31</td>
<td>J1012+5307</td>
<td>1.64 ±0.12</td>
<td>50</td>
</tr>
<tr>
<td>J1713+0747b</td>
<td>1.53 ±0.06</td>
<td>51</td>
<td>B1802-07d</td>
<td>1.26 +0.08</td>
<td>31</td>
</tr>
<tr>
<td>B1855+49b</td>
<td>1.57 ±0.12</td>
<td>52</td>
<td>J0621+1002</td>
<td>1.70 ±0.10</td>
<td>53</td>
</tr>
<tr>
<td>J0751+1807</td>
<td>1.26 ±0.11</td>
<td>53</td>
<td>J0437+4715</td>
<td>1.76 ±0.20</td>
<td>54</td>
</tr>
<tr>
<td>J1141-6545</td>
<td>1.27 ±0.01</td>
<td>55</td>
<td>J1748-2446d</td>
<td>1.91 ±0.02</td>
<td>56</td>
</tr>
<tr>
<td>J1748-2446d</td>
<td>1.79 ±0.10</td>
<td>56</td>
<td>J1909-3744d</td>
<td>1.47 ±0.03</td>
<td>57</td>
</tr>
<tr>
<td>J0024-7204H</td>
<td>1.48 +0.06</td>
<td>57</td>
<td>B1802-2124</td>
<td>1.24 ±0.11</td>
<td>58</td>
</tr>
<tr>
<td>J0514-4002A</td>
<td>1.49 ±0.04</td>
<td>58</td>
<td>B1516+02Bd</td>
<td>2.08 ±0.19</td>
<td>59</td>
</tr>
<tr>
<td>J1748-2021B</td>
<td>2.74 ±0.21</td>
<td>60</td>
<td>J1750-37Ad</td>
<td>1.26 ±0.19</td>
<td>60</td>
</tr>
<tr>
<td>J1738+0333</td>
<td>1.55 ±0.05</td>
<td>61</td>
<td>B1911-5958Ad</td>
<td>1.34 ±0.08</td>
<td>61</td>
</tr>
<tr>
<td>J1614-2230</td>
<td>1.97 ±0.04</td>
<td>64</td>
<td>J2043+1711c</td>
<td>1.85 ±0.15</td>
<td>65</td>
</tr>
<tr>
<td>J1910+1256c</td>
<td>1.67 ±0.06</td>
<td>66</td>
<td>J2106+1948c</td>
<td>1.03 ±0.1</td>
<td>66</td>
</tr>
<tr>
<td>J1853+1303c</td>
<td>1.47 ±0.05</td>
<td>67</td>
<td>J1045-4509</td>
<td>1.19 ±0.20</td>
<td>31</td>
</tr>
<tr>
<td>J1804-2718</td>
<td>1.39 ±0.04</td>
<td>68</td>
<td>J2019+2425</td>
<td>1.205 ±0.005</td>
<td>66</td>
</tr>
</tbody>
</table>

Neutron star-main sequence binaries

| J0045-7319     | 1.58 ±0.34      | 31        | J1903+0327c     | 1.667 ±0.021     | 67        |

---

*Black hole due to lack of pulsations?

Companion masses are from Reference 31.

Binary period-white dwarf masses are from Reference 26.

Globular cluster binary.

3-σ error.
once chemical potentials of nucleons $\mu_N$ exceed rest mass energies, new particle species can appear.

generally: appearance of new particle species "softens" the equation of state, i.e. reduces the pressure, i.e. reduces the maximum possible mass.
Mass function

• these quantities can be combined into the “mass function”

\[ f = \frac{Pv_1}{2\pi G} \]

• importance of the mass function:
  • \( f \) has the dimension of a mass
  • \( f \) is a lower limit on the mass of the companion star!
Mass function

• example:

• from spectrum of one star $P$ and $v_1$ are measured and yield

$$f = \frac{Pv_1}{2\pi G} = 5.5 \ M_\odot$$

• this means that the unobserved companion has at least 5.5 solar masses

• this is used to identify “black hole candidates”

neutron stars cannot have masses larger than 3 solar masses

compact objects with more than 3 solar masses must be black holes
Fig. 1.—Remnants of massive single stars as a function of initial metallicity (y-axis; qualitatively) and initial mass (x-axis). The thick green line separates the regimes where the stars keep their hydrogen envelope (left and lower right) from those where the hydrogen envelope is lost (upper right and small strip at the bottom between 100 and 140 $M_\odot$). The dashed blue line indicates the border of the regime of direct black hole formation (black). This domain is interrupted by a strip of pair-instability supernovae that leave no remnant (white). Outside the direct black hole regime, at lower mass and higher metallicity, follows the regime of BH formation by fallback (red cross-hatching and bordered by a black dot-dashed line). Outside of this, green cross-hatching indicates the formation of neutron stars. The lowest mass neutron stars may be made by O/Ne/Mg core collapse instead of iron core collapse (vertical dot-dashed lines at the left). At even lower mass, the cores do not collapse and only white dwarfs are made (white strip at the very left).
Fig. 2.—Supernovae types of nonrotating massive single stars as a function of initial metallicity and initial mass. The lines have the same meaning as in Fig. 1. Green horizontal hatching indicates the domain where Type IIp supernovae occur. At the high-mass end of the regime they may be weak and observationally faint because of fallback of $^{56}$Ni. These weak SN Type IIp should preferentially occur at low metallicity. At the upper right-hand edge of the SN Type II regime, close to the green line of loss of the hydrogen envelope, Type IIb supernovae that have a hydrogen envelope of $\leq 2 M_\odot$ are made (purple cross-hatching). In the upper right-hand quarter of the figure, above both the lines of hydrogen envelope loss and direct black hole formation, Type Ib/c supernovae occur; in the lower part of their regime (middle of the right half of the figure) they may be weak and observationally faint because of fallback of $^{56}$Ni, similar to the weak Type IIp SNe. In the direct black hole regime no “normal” (non-jet-powered) supernovae occur since no SN shock is launched. An exception are pulsational pair-instability supernovae (lower right-hand corner, brown diagonal hatching) that launch their ejection before the core collapses. Below and to the right of this we find the (nonpulsational) pair-instability supernovae (red cross-hatching), making no remnant, and finally another domain where black hole are formed promptly at the lowest metallicities and highest masses (white) where no SNe are made. White dwarfs also do not make supernovae (white strip at the very left).