Gravitational Astronomy with Advanced and 3G Detectors: Prospects and Challenges

NORDITA Winter School, January 8-10, Stockholm, Sweden

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School of Physics and Astronomy, Cardiff University, UK
Advanced LV Primary Science Objectives

- Make a direct detection of gravitational waves
  - Binary neutron stars are the most likely sources
- Constrain the rate of compact binary coalescences
  - Current rates are uncertain by 3 orders of magnitude. Advanced detectors will measure the rate to within a factor of a few in the local (<500 Mpc) Universe
- Detect binary black holes and neutron star-black hole binaries
  - First ever detection of such systems - a new population of astronomical sources
- Verify GRB-GW Association
  - Check if compact binaries in which at least one of the companions is a neutron star are progenitors of short gamma-ray bursts
- Measure the Hubble parameter to within 5%
  - Compact binary inspirals are self-calibrating standard sirens
  - To be useful as standard candles it is necessary to identify the host and measure its redshift and so sky localization is a key science objective
  - If we binary black holes occur in sufficient numbers then EM identification may not be necessary but it would still be necessary to have a good resolution
- Carry out strong field tests of general relativity
What will ET observe and what can it tell?

- Observe radiation from
  - BH collisions when the Universe was still in its infancy assembling the first galaxies
  - NS collisions when star formation in the Universe was at its peak
  - Formation of black holes and neutron stars in supernovae and collapsars in the local neighbourhood
  - Might observe stochastic backgrounds of cosmological and astrophysical origin

- LV and ET could provide new insights into
  - The secret births and lives of black holes and neutron stars, their demographics, populations and their masses and spins
  - Hubble parameter, dark energy and its variation with redshift
  - Equation-of-state of matter at supra-nuclear densities
  - Early history of the Universe’s evolution
Horizon distance for compact binary mergers

- Horizon distance: Distance in Mpc at which one Advanced LIGO detector can see an optimally-located, optimally oriented binary merger with an SNR=8 as a function of total mass.
- Averaging over sky location and orientation degrades this distance (Mpc).
- Experience from Initial LIGO-Virgo suggests that reliable detection is possible with an average SNR/detector ~ 8.

Important to use the right templates, including IMR, and spin effects!

Inspiral-merger-ringdown

EOB template (light-ring)

Ringdown template

Inspiral-only template (ISCO)

P. Ajith (2009)
ET Distance Reach for Compact Binary Mergers

Slide Curtsey: L. Santamaria

Tuesday, 15 January 2013
TOPICAL REVIEW

Predictions for the rates of compact binary coalescences observable by ground-based gravitational-wave detectors

Table 5. Detection rates for compact binary coalescence sources.

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<th>IFO</th>
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Virgo’s Antenna Pattern
ET's Antenna Pattern

Colatitude $\theta$ (in radians)

Azimuth $\phi$ (in radians)
Baselines in light travel time (ms)
Sky Localization Error Ellipses

Red crosses denote regions where the network has blind spots

Fairhurst 2011

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Sky Localization Error Ellipses

Fairhurst 2011
Sky Localization

Sky Localization (\(\Delta \Omega\) in sq deg) at 90% confidence.

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Large survey telescopes like LSST can scan a 10 deg x 10 deg sky patch in 30 min.
Luminosity Distance

Probability

Percentage Error in Luminosity Distance (100 $\Delta D_L/D_L$)

- HIL
- HIV
- HLV
- ILV
- HHLV
- HILV
- HALV
Orientation of the Binary

Error in orientation of the orbit ($\Delta \cos \iota$)

Probability

Error in orientation of the orbit ($\Delta \cos \iota$)

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Fundamental Physics

- Properties of gravitational waves
  - Testing GR beyond the quadrupole formula
    - Binary pulsars consistent with quadrupole formula; they don’t measure properties of GW
  - How many polarizations are there?
    - In Einstein’s theory only two polarizations; a scalar-tensor theory could have six
  - Do gravitational waves travel at the speed of light?
    - There are strong motivations from string theory to consider massive gravitons
    - Binary pulsars constrain the speed to few parts in a thousand
    - GW observations can constrain to 1 part in $10^{18}$

- EoS of dark energy
  - Black hole binaries are standard candles/sirens

- EoS of supra-nuclear matter
  - Signature of EoS in GW emitted when neutron stars merge

- Black hole no-hair theorem and cosmic censorship
  - Are BH (candidates) of nature BH of general relativity?

- An independent constraint/measurement of neutrino mass
  - Delay in the arrival times of neutrinos and gravitational waves

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Testing Black Hole No-Hair Theorem

- Deformed black holes are unstable; they emit energy in their deformation as gravitational waves
- Superposition of damped waves with many different frequencies and decay times
- In Einstein’s theory, frequencies and decay times all depend only on the mass $M$ and spin $j$ of the black hole
- Measuring two or more modes would constrain Einstein’s theory or provide a smoking gun evidence of black holes
- If modes depend on other parameters (e.g., the structure of the central object), then test of the consistency between different mode frequencies and damping times would fail
- The amplitude of the modes carry additional information about what caused the deformity

BBH Signals as Testbeds for GR

- Gravity gets ultra-strong during a BBH merger compared to any observations in the solar system or in binary pulsars
  - In the solar system: $\phi/c^2 \sim 10^{-6}$
  - In a radio binary pulsar it is still very small: $\phi/c^2 \sim 10^{-4}$
  - Near a black hole $\phi/c^2 \sim 1$
- Merging binary black holes are the best systems for strong-field tests of GR
- Dissipative predictions of gravity are not even tested at the 1PN level
  - In binary black holes even $(v/c)^7$ PN terms will not be adequate for high-SNR ($\sim 100$) events
Testing GR by observing non-linear effects

- Binary inspiral waveform depends on many post-Newtonian coefficients
  - $\Psi_0, \Psi_2, \Psi_3, ...$
  - They correspond to different physical effects, e.g. GW tails
- In the case of non-spinning binaries $\Psi_0, \Psi_2, \Psi_3, ...$ depend on just the two masses $m_1$ and $m_2$
- By assuming they are all independent one can check to see if GR is the correct theory

Blanchet and Schaefer (1994)
Black Holes Sing
Their Past
Black Hole Perturbation Theory
Black Hole Perturbation Theory

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**Quasi-normal modes are damped sinusoids**

- Far away from the source the waveform emitted by a perturbed black hole has the form

\[ h(t) = A \frac{M}{r} \exp(-t/\tau) \cos(\omega t + \varphi_0) \]

† Amplitude \( A \) depends on the nature of perturbation

† \( r \) is the distance to the black hole

† \( \omega \) and \( \tau \) are the mode frequency and damping time
Black Hole No-Hair Theorem $\omega_{nlm}, \tau_{nlm}$
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- Absence of quasi-normal modes after merger might indicate failure of GR or existence of naked singularities
Typical Values of the Dominant Mode

- Mode frequencies are inversely proportional to BH mass and decay times directly proportional to it.
- Gravitational waves being quadrupolar the most dominant mode excited is $l = 2$.
- The frequency and the decay time of the 22 mode (i.e. $l=2, m=2$) is

$$f = 1.2 \text{ Hz } \left(\frac{10 M_\odot}{M}\right)$$

$$\tau = 0.55 \text{ ms } \left(\frac{M}{10 M_\odot}\right)$$

$$Q = \frac{1}{2} \tau \omega \sim 2$$

$$f = 2.0 \text{ kHz and } Q = 5 \text{ for } j = 0.9$$
Can Quasi-normal Modes Reveal Their Perturber?

- If mode frequencies depend only on the black hole’s mass and spin how can they reveal what caused the perturbation?
- No-hair theorem really doesn’t apply to deformed BHs
- Should be possible to measure not just BH mass and spin but also, for instance, the mass ratio of the progenitor binary from the QNMs produced in the aftermath of merger
- The key is that the amplitude of the modes carry additional information
- They depend on the nature of the perturber

\[ h^+_{lm} - i h^\times_{lm} = \frac{A_{lm} M}{r} e^{i \omega_{lm} t} e^{-t/\tau_{lm}} \]

- If we have observe only one mode then the amplitude would be degenerate with other parameters - distance to the black hole, its location on the sky, etc.
- Observing higher order modes should help break the degeneracy
Cosmology

Cosmography
- Build the cosmic distance ladder, strengthen existing calibrations at high $z$
- Measure the Hubble parameter, dark matter and dark energy densities, dark energy EoS $w$, variation of $w$ with $z$

Black hole seeds
- Black hole seeds could be intermediate mass black holes
- Might explore hierarchical growth of central engines of black holes

Dipole anisotropy in the Hubble parameter
- The Hubble parameter will be “slightly” different in different directions due to the local flow of our galaxy

Anisotropic cosmologies
- In an anisotropic Universe the distribution of $H$ on the sky should show residual quadrupole and higher-order anisotropies

Primordial gravitational waves
- Quantum fluctuations in the early Universe could produce a stochastic b/g

Production of GW during early Universe phase transitions
- Phase transitions, pre-heating, re-heating, etc., could produce detectable stochastic GW
Gravity’s Standard Sirens

* To measure the luminosity distance to a source we need its **apparent** and **absolute** luminosities

* Gravitational wave observations of compact binary inspirals can measure both
  * **Apparent luminosity** this is GW strain in our detector
  * **Absolute luminosity** this is the rate at which frequency changes

* Therefore, binary black hole inspirals are **self-calibrating standard sirens**

* However, GW observations alone cannot determine the redshift to a source

* Joint gravitational-wave and optical observations can facilitate a **new cosmological tool**
is further augmented by a factor of 1.12. At this rate, we find that one year of observation should be enough to measure \( H_0 \) to an accuracy of \( \sim 1\% \) if SHBs are dominated by beamed NS-BH binaries using the “full” network of LIGO, Virgo, AIGO, and LCGT—admittedly,
Hubble Constant from Advanced Detectors
without EM counterparts

- **25 events:**
  - $H_0 = 69 \pm 3$ km s$^{-1}$ Mpc$^{-1}$ (~4% at 95% confidence)

- **50 events:**
  - $H_0 = 69 \pm 2$ km s$^{-1}$ Mpc$^{-1}$ (~3% at 95% confidence)

- **WMAP7+BAO+SnIa (Komatsu et al., 2011):**
  - $H_0 = 70.2 \pm 1.4$ km s$^{-1}$ Mpc$^{-1}$ (~2% at 68% confidence)
The error bars correspond to the posterior median value obtained from 20 realisations of 50 GW sources. The dots correspond to the number of events considered in the analysis.
**Figure 3.** Scatter plot of the retrieved values for $(\Omega_\Lambda, w)$, with 1-$\sigma$, 2-$\sigma$ and 3-$\sigma$ contours, in the case where weak lensing is not corrected.
Measuring $w$ and its variation with $z$

$$w(z) \equiv \frac{p_{de}}{\rho_{de}} = w_0 + w_a z / (1 + z)$$

Baskaran, Van Den Broeck, Zhao, Li, 2011
Measuring a cosmological distance–redshift relationship using only gravitational wave observations of binary neutron star coalescences

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J. Read  
Department of Physics and Astronomy, The University of Mississippi, P.O. Box 1848, Oxford, Mississippi 38677-1848

Hubble without the Hubble:  
Cosmology using advanced gravitational-wave detectors alone

Stephen R. Taylor*  
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NSF Astronomy and Astrophysics Postdoctoral Fellow,  
MIT Kavli Institute, Cambridge, MA 02139; and  
School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham, B15 2TT  
(Dated: January 31, 2012)
Messenger-Read Method: Make use of the post-Newtonian Tidal Term


\[ \Psi_{PP}(f) = 2\pi ft_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta} x^{5/2} \sum_{k=0}^{N} \alpha_k x^{k/2} \]


\[ \Psi_{\text{tidal}}(f) = \sum_{a=1,2} \frac{3\lambda_a}{128\eta} \left[ -\frac{24}{\chi_a} \left( 1 + \frac{11\eta}{\chi_a} \right) \frac{x^{5/2}}{M^5} \right. \]

\[ \left. -\frac{5}{28\chi_a} \left( 3179 - 919\chi_a - 2286\chi_a^2 + 260\chi_a^3 \right) \frac{x^{7/2}}{M^5} \right] \]

\[ x = (\pi Mf)^{2/3} \quad \lambda = (2/3)R_{ns}^5 k_2 \]
Measurement accuracy of source redshift

\[ \frac{\Delta z}{z} \]

\[ 1 \]

\[ 10^{-1} \]

\[ 10^{-2} \]

\[ \text{redshift } z \]

\[ 0.01 \]

\[ 0.1 \]

\[ 1 \]

APR

SLY

MS1

\[ \text{Messenger and Read, PRL, 2011} \]
Distribution of Chirp Mass

\[ M \sim N(\mu_c, \sigma_c^2), \]

\[ \mu_c \approx 2(0.25)^{3/5} \mu_{NS}, \quad \sigma_c \approx \sqrt{2}(0.25)^{3/5} \sigma_{NS}, \]

\[ \mu_{NS} \in [1.0, 1.5] M_\odot, \quad \sigma_{NS} \in [0, 0.3] M_\odot \]

\[ w(a) = w_0 + w_a (1 - a), \]

\[ w(z) = w_0 + w_a \left( \frac{z}{1 + z} \right). \]
Initially small black holes may grow by hierarchical merger

- ET could observe seed black holes if they are of order 1000 solar mass
ET Distance Reach for Compact Binary Mergers

Slide Curtsey: L. Santamaria
BH mass correlated with Galaxy luminosity and bulge mass

GÜLTEKIN ET AL.
BHs grow by merger

Mayer et al, Science 2007, 316, 1874
Merger Tree Simulations Predict Frequent Mergers

2.4 Astrophysical Black Holes

Figure 2fij2fi: A state-of-art hydrodynamical simulation by D M (visualising the cosmic evolution of the baryonic density field and of their embedded black holes, in the $\Lambda$CDM cosmology. Each panel shows the same region of space ($3.75\ h^{-1} Mpc$ on a side) at different redshift, as labelled. The circles mark the positions of the black holes, with a size that encodes the mass, as indicated in the top left panel (numerical force resolution limits the lowest black hole mass to $10^5 M_\odot$). The projected baryonic density field is colour-coded with brightness proportional to the logarithm of the gas surface density. The images show that the black holes emerge in halos starting at high redshift (as early as $z \sim 10$) and later grow by gas inflows that accompany the hierarchical build-up of ever larger halos through merging. As the simulation evolves, the number of black holes rapidly increases and larger halos host increasingly larger black holes. No black holes as massive as $10^9 M_\odot$ are present in the simulated box because they are extremely rare.

Figure 2fij3fi: A cartoon of the merger-tree history for the assembly of a galaxy and its central black hole, through the mergers of smaller galaxies and the coalescences of their black holes.

Massive black holes in the cosmological framework

Black holes are expected to transit into the mass interval to which NGO is sensitive along the course of their cosmic evolution NGO will then map and mark the loci where galaxies form and cluster) using black holes as clean tracers of their assembly by capturing gravitational waves emitted during their coalescence) that travelled undisturbed from the sites where they originatedfi These key findings hint in favour of the existence) at any redshift) of an underlying population of black holes of a smaller variety) with masses of $10^4 M_\odot$–$10^7 M_\odot$ that grew in mass along cosmic histories inside their galaxies) through episodes of merging and accretionfi The evolution of black holes mimics closely that of their host galaxies within the currently favoured cosmological paradigm: a universe dominated by cold dark matter, CDM-fi

Observations show that the mass content of the universe is dominated by CDM) with baryons contributing only at a $10^3$ level to the CDM) and that the spectrum of primordial density perturbations contains more power at lower masses ($M < 10^{9.5}$). Thus) at the earliest epoch) the universe was dominated by small density perturbationsfi Regions with higher density grow in time) to the point they decouple from the Hubble flow and collapse and virialise forming self-gravitating halosfi The first objects that collapse under their own self%gravity are small halos that grow bigger through mergers with other halos and accretion of surrounding matterfi This is a bottom Tuesday, 15 January 2013
This latter stage can be, again, modelled analytically in terms of black hole perturbation theory—At the end of the inspiral, the black holes velocities approach enough so that they can be treated analytically as point particles within the post-Newtonian 2PN3 approximation process in which the black holes spiral together on near-circular orbits—The black holes have a separation wide in which the black holes plunge and merge together, forming a highly distorted, perturbed remnant—At the end of the merger, the ring-down the final black hole is left in a quiescent state, with no residual structure besides its Kerr spacetime—The full waveform is computationally very demanding even for few orbital cycles—Full waveforms can be designed by stitching together the analytic merger and ring-down phases—In the following estimations we will mostly employ phenomenological descriptions of how the ring-down evolves—SNR is computed using PhenomC waveforms inclusive of the inspiral and the ring-down forms—The gravitational waves emitted by black hole binaries with total mass \( M > 10^4 \) \( M_\odot \) in the source rest frame3 as small as \( 10^{-5} \) in the rest frame of the source—SNR is measured to an unprecedented accuracy, up to the \( 50 \) level, whereas absolute errors that includes the uncertainties on the redshift parameter space relevant for addressing scientific questions on the evolution of the black hole population—NGO Can Detect MBBH Mergers to \( z = 20 \)
BBH Mergers in NGO are Loud:
Enables accurate measurement of masses and spins

- Masses can be measured to an accuracy of 0.1% to 1%
- Absolute errors in dimensionless spin in the range 0.01 to 0.1
- For sources within $z=1$ distance could be measured to within 1-10%

![Graph showing mass-to-light ratio for different redshifts](Image)
A brief history of the Universe

- CMB $f < 3 \times 10^{-17}$ Hz probes $300,000$ yrs $< t_e < 14$ Gyr
- Pulsars $f \sim 10^{-8}$ Hz probe $t_e \sim 10^{-4}$ s ($T \sim 50$ MeV)
- LISA $f \sim 10^{-3}$ Hz probes $t_e \sim 10^{-14}$ s ($T \sim 10$ TeV)
- ET $f \sim 10$ Hz probes $t_e \sim 10^{-20}$ s ($T \sim 10^6$ GeV)
- LIGO $f \sim 100$ Hz probes $t_e \sim 10^{-24}$ s ($T \sim 10^8$ GeV)
- (Planck scale $f \sim 10^{11}$ Hz has $t_e \sim 10^{-43}$ s ($T \sim 10^{19}$ GeV)
Stochastic Backgrounds

- Primordial background
  - Quantum fluctuations produce a background GW that is amplified by the background gravitational field

- Phase transitions in the Early Universe
  - Cosmic strings - kinks can form and “break” producing a burst of gravitational waves
Primordial Backgrounds in ET

Cosmological energy density in GW, Ω\(_{gw}(f)\):

- Cosmic strings (p=1, ε=1)
- SUSY flat direction (1)
- SUSY flat direction (2)
- Tachyonic preheating
- Inflation (r=0.15, n\(_T\)=0.2)
- SUSY phase transition, F\(^{1/2}\)=10\(^6\) GeV

Frequency (Hz):

ET-D

ET-B

AdvLIGO

G\(\mu\)=10\(^{-6}\)

G\(\mu\)=10\(^{-9}\)

Tuesday, 15 January 2013
Astrophysics

- Unveiling progenitors of short-hard GRBs
  - Understand the demographics and different classes of short-hard GRBs
- Understanding Supernovae
  - Astrophysics of gravitational collapse and accompanying supernova?
- Evolutionary paths of compact binaries
  - Evolution of compact binaries involves complex astrophysics
    - Initial mass function, stellar winds, kicks from supernova, common envelope phase
- Finding why pulsars glitch and magnetars flare
  - What causes sudden excursions in pulsar spin frequencies and what is behind ultra high-energy transients of EM radiation in magnetars
    - Could reveal the composition and structure of neutron star cores
- Ellipticity of neutron stars as small as 1 part in a billion (10μm)
  - Mountains of what size can be supported on neutron stars?
- NS spin frequencies in LMXBs
  - Why are spin frequencies of neutron stars in low-mass X-ray binaries bounded?
- Onset/evolution of relativistic instabilities
  - CFS instability and r-modes
Measuring the Mass Function of Neutron Stars

% error in neutron star mass (100 \Delta m/m)

Companion mass in M_\odot

\% error in neutron star mass (100 \Delta m/m) vs Companion mass in M_\odot

Slide from Van Den Broeck

Tuesday, 15 January 2013
Inferring the History of Star Formation Rate

![Graph showing the history of star formation rate across different redshifts.

Underlying and recovered rate / $10^3$

Redshift $z$

Slide from Van Den Broeck

Tuesday, 15 January 2013

Hopkins and Beacom
Fardal et al.
Wilkins et al.
Nagamine et al.

- Black solid line: Hopkins and Beacom
- Red dotted line: Fardal et al.
- Green dashed line: Wilkins et al.
- Blue dotted line: Nagamine et al.
Unveiling the Origin of GRBs

- ET can detect model-independent radiation from collapsars if $E_{GW} > 5\% \, M_\odot$
- Soft Gamma Repeaters could be seen both in the Milky Way and the local neighbourhood provided if $E_{GW} > 10^{-8} \, M_\odot$

Slides from Clark and Sutton

Tuesday, 15 January 2013
Mountains on Neutron Stars

- ET will check if neutron stars (10 km in radius) have mountains that are smaller than 10 micro meters
- This could constrain models about their crustal strengths
Neutron star mergers and equation of state of neutron stars

- Spectrum of gravitational radiation from black hole binaries is featureless (that's why they are standard candles)
- Radiation from binary neutron star mergers carries an imprint of the star's mass and equation of state

Andersson et al. 2011
Supernovae

- Standard candles of astronomy
  - Our knowledge of the expansion rate of the Universe at redshift of $z=1$ comes from SNe
- Produce dust and affect evolution of galaxies
  - Heavy elements are only produced in SNe
- They are precursors to formation of neutron stars and black holes
  - The most compact objects in the Universe
- SNe cores are laboratories of complex physical phenomena
  - Most branches of physics and astrophysics needed in modelling
    - General relativity, nuclear physics, relativistic magnetohydrodynamics, turbulence, neutrino viscosity and transport, ...
- Unsolved problem: what is the mechanism of shock revival?
Core Collapse SNe

- **Energy reservoir**
  - few x $10^{53}$ erg

- **Explosion energy**
  - $10^{51}$ erg

- **Time frame for explosion**
  - 300 - 1500 ms after bounce

- **Formation of black hole**
  - At baryonic mass > 1.8-2.5 M

Slide from Ott

- SN Explosion
- Shock Revival
- No Shock Revival
- BH formation
  - "Collapsar"

Evolved Massive Star

Iron Core Collapse

Protoneutron Star, Stalled Shock, Accretion
Accretion Induced Collapse

- Collapse of accreting, probably rotating White Dwarfs
  - Neutrino-driven or magneto-rotational explosion
  - Explosion probably weak, sub-luminous

- Might not be seen in optical
- Potential birth site of magnetars - highly (10^{15}-10^{16} G) magnetized neutron stars

Slide from Ott

[Image of a diagram showing accretion, white dwarf, accretion-induced collapse, protoneutron star, shock, and SN explosion]
**SNe Rate in ET**

- ET sensitive to SNe up to 5 Mpc
- Could observe one SN once in few years
- Coincident observation with neutrino detectors
- Might be allow measurement of neutrino mass
- Plots show the spectra of SNe at 10 Kpc for two different models

**Slide from Ott**

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**Convection/SASI/Neutrino Mechanism**

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**Rotating Collapse / MHD**

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Tuesday, 15 January 2013
Pulsar Glitches

- Pulsars have fairly stable rotation rates:
  - However, observe the secular increase in pulse period
- Glitches are sudden dips in the rotation period
  - Vela shows glitches once every few years
- Could be the result of transfer of angular momentum from core to crust
  - At some critical lag rotation rate of superfluid core couples to the crust, imparting energy to the crust

\[
\begin{align*}
\Delta J &\sim I_* \Delta \Omega \\
\Delta \Omega / \Omega &\sim 10^{-6} \\
\Delta E &\sim 10^{-13} - 10^{-11} M_\odot c^2
\end{align*}
\]

A glitch in Vela

A composite Vela image

Friday, 19 June 2009

Amaldi 09
J Clark, June 2009
LIGO-G0900574-v1
NS Normal Mode Oscillations

- Sudden jolt due to a glitch, and superfluid vortex unpinning, could cause oscillations of the core, emitting gravitational waves.
- These normal mode oscillations have characteristic frequencies and damping times that depend on the equation-of-state.
- Detecting and measuring normal modes could reveal the equation-of-state of neutron stars and their internal structure.

Slide from Melatos

![Diagram of neutron star with labels and equations]

\[ \Omega_{\text{lattice}} \]

\[ F_{\text{Magnus}} \]

\[ \Omega_{sf} - \Omega_{\text{lattice}} \]

\[ F_{\text{Magnus}} \]

\[ \Omega_{\text{lattice}} \]

\[ \text{f-mode Frequency} \]

\[ \text{f-mode Damping Time} \]

\[ \text{M}_{\text{NS}} = 1.2 \text{M}_{\odot} \]

\[ \text{M}_{\text{NS}} = 1.4 \text{M}_{\odot} \]

\[ \text{M}_{\text{NS}} = 1.5 \text{M}_{\odot} \]

\[ \text{M}_{\text{NS}} = 1.8 \text{M}_{\odot} \]

\[ \text{M}_{\text{NS}} = 2.0 \text{M}_{\odot} \]

Tuesday, 15 January 2013
Accreting Neutron Stars

• Spin frequencies of accreting NS seems to be stalled below 700 Hz
  • Well below the break-up speed
• What could be the reason for this stall?
  • Balance of accretion torque with GW back reaction torque
• Could be explained if ellipticity is $\sim 10^{-8}$
  • Could be induced by mountains or relativistic instabilities, e.g. r-modes

Slide from Melatos
Challenges

- **Reliable estimation of the background rate**
  - Current demands on detection confidence are far too high
    - A false alarm rate (FAR) of 1 per several thousand years
  - New algorithms are needed so that we can make a detection with a FAR of 1 per 100 years

- **Developing waveforms that are as close to GR prediction as possible**
  - Precession due to spins, higher order modes, inspiral-merger-ringdown
    - Numerical relativity is still far away from producing on-the-fly template waveforms

- **Handling longer waveforms**
  - In LV “Transient” signals could last for 30 minutes are longer

- **Faster algorithms for parameter estimation**
  - Nested sampling and multi-nest algorithms are already effective but they are still not fast enough
  - The event rate could be as high as several per day
Summary

Gravitational wave detectors can potential impact...

**Fundamental Physics**
- Is the nature of gravitational radiation as predicted by Einstein?
- Is Einstein theory the correct theory of gravity?
- Are black holes in nature black holes of GR?
- Are there naked singularities?

**Astrophysics**
- What is the nature of gravitational collapse?
- What is the origin of gamma ray bursts?
- What is the structure of neutron stars and other compact objects?

**Cosmology**
- How did massive black holes at galactic nuclei form and evolve?
- What is dark energy?
- What phase transitions took place in the early Universe?
- What were the physical conditions at the big bang?
How can we test non-linear effects?

• If Einstein’s theory is a correct description of gravity, then the phasing of the waveform is given by

\[ \psi_i(M, \eta) = M^{-5/3} f_i(\eta) (\pi M \eta^{-3/5})^{(i-5)/3} \]

• Deviations from GR can be phenomenologically modelled as

\[ \psi_{1GR}(M, \eta) = 0 \rightarrow M^{-5/3} (\pi M \eta^{-3/5})^{-4/3} \delta \chi_1, \]
\[ \psi_{2GR}(M, \eta) \rightarrow M^{-5/3} f_2(\eta) (\pi M \eta^{-3/5})^{-1} [1 + \delta \chi_2], \]
\[ \psi_{3GR}(M, \eta) \rightarrow M^{-5/3} f_3(\eta) (\pi M \eta^{-3/5})^{-2/3} [1 + \delta \chi_3]. \]

Use Bayesian Inference to Test GR

1. Example: A GR injection

![Graph showing posterior PDFs for a GR injection](Image 1)

Let us first look at a GR source with $f$.

The Bayes factors for the various contributions are

The GR hypothesis is strongly disfavored already at SNR for the various

FIGw 18: The cumulative Bayes factors against the GR hypothesis

Let us now look at some posterior PDFs. In Fig., $\theta$ and $\delta \chi$ with

$\theta$ and $\delta \chi$ respectively as free parameters;

We see that the distributions are all narrowly centered on zero.

FIGw 19: Posterior PDFs for a GR injection.

Top: measured with a waveform that has

$\eta$ and $\delta \chi$ respectively $\theta$ and $\delta \chi$ with

$\theta$ and $\delta \chi$ respectively as free parameters;

$\delta \chi$ and $\delta \chi$ with

$\theta$ and $\delta \chi$ respectively as free parameters;

The Bayes factors for the various.

We can also look at the

Let us first look at a GR source with $f$.

The Bayes factors for the various.

Hence we start with an analysis of

FIGw 18: The cumulative Bayes factors against the GR hypothesis

Let us now look at some posterior PDFs. In Fig., $\theta$ and $\delta \chi$ with

$\theta$ and $\delta \chi$ respectively as free parameters;

We see that the distributions are all narrowly centered on zero.

FIGw 19: Posterior PDFs for a GR injection.

Top: measured with a waveform that has

$\eta$ and $\delta \chi$ respectively.

Standard deviations of $\eta_1$, $\eta_2$, and $\eta_3$ respectively.

In each case the distribution is tightly centered on zero.

$\delta \chi_1$, $\delta \chi_2$, and $\delta \chi_3$ respectively.
2. Example: A signal with $\delta \chi_3 = 0.1$. 

![PDF plot with peak at the correct value](image)
Figure 4.

Lefty normalized distribution of log odds ratios for individual sources where the signals are in accordance with $H_{GR}$ blue dotted or have a deviation of the form given in Eq.m $f_{GR}$ red striped. Righty normalized distribution of logs of the combined odds ratios for the same signals as at the top but randomly arranged in catalogues of $pt$ sources each. The effect of combining sources is in this case profound. Only a small difference between background and foreground is visible when considering individual sources. For catalogues of $pt$ sources, the differentiation becomes significant.

To characterise such a confidence, we introduce the concept of efficiency. Assume one has two distributions of odds ratios. The distribution of odds ratio obtained when the simulated signals, collectively denoted by $\kappa$, are in agreement with $H_{GR}$ $P_{mod\{GR\}}|\kappa, H_{GR}, I_g$, and the distribution obtained when the simulated signals $\kappa'$ adhere to some alternative theory, $P_{mod\{GR\}}|\kappa', H_{alt}, I_g$. We then look at the portion of the latter that lies beyond a portion $\beta$ of the former. To be more precise, the efficiency $\zeta_k$ is given by:

$$\zeta_k = \frac{\int_{-\infty}^{\beta} P_{mod|\kappa', H_{alt}, I_g}}{\int_{-\infty}^{\infty} P_{mod|\kappa, H_{GR}, I_g}}$$

where $P_{mod|\kappa, H_{GR}, I_g}$ is determined by a fixed false alarm probability, $p_{\beta}$ where $\beta = \int_{-\infty}^{0} P_{mod|\kappa, H_{GR}, I_g}$.

In Fig.m $tk$ we show the efficiency for the example shown in subsection $rmq$ as a function of the catalogue size, for $\beta \in \{0.01, 0.05, 0.08\}$. Which sources are placed together in a catalogue is determined randomly. To understand the statistical fluctuations in the efficiency when collecting sources into catalogues in different ways, for the same set of signals we considered tooo random orderings in which the signals are combined into catalogues. The resulting median and the $uwc$ confidence levels are shown as the central curve and the error bars, respectively.

As can be seen in Fig.m $tk$ the acceptance probability rises sharply as a function of the catalogue size. This underscores the importance of considering all the detected source in a coherent fashion, as was explained in subsection $qms$. Even though a single detection might not yield confidence in a deviation from $GR$, coherently adding information from multiple sources can rapidly increase this confidence.

To put the numbers in Fig.m $tk$ into perspective, the predicted rate for binary inspiral in the so-called 'realistic' case is so per year.

4. Conclusions and Discussion

We have given two striking examples to support the claim that the method proposed in $ts$ can distinguish deviations that are not captured by the limited model waveforms, as long as the Combining Multiple Observations: 15 Catalogues