The total negative charge:

\[ \Phi = \int_{0}^{\infty} g(r) 4\pi r^2 \, dr \]

\[ = -C \int_{0}^{\infty} e^{-2r/a} 4\pi r^2 \, dr \quad \frac{r}{a} = \xi \]

\[ = -C \left[ 4\pi \right] e^{-2\xi} a^2 \int_{0}^{\infty} d\xi \]

\[ = -C \left[ 4\pi a^3 \int_{0}^{\infty} e^{-2\xi} \, d\xi \right] \]

\[ = -4\pi a^3 C \left[ \frac{e^{-2\xi}}{-2} \right]_{0}^{\infty} \]

\[ = -4\pi a^3 C \left[ \frac{e^{-2\xi}}{-2} \right]_{0}^{\infty} \]

\[ = -4\pi a^3 C \left[ \frac{1}{2} \left( e^{-2\xi} \right) \right]_{0}^{\infty} \]

\[ = 4\pi a^3 C \left[ \frac{1}{2} \left( e^{-2\xi} \right) \right]_{0}^{\infty} \]

As total charge must be zero, \( \Phi = -e \)

\[ \Rightarrow \quad C = -\frac{e}{4\pi a^3} \]

\[ C = \frac{e}{4\pi a^3} \]
(b) The total electric charge inside a volume of radius \( R \) is

\[
\Phi_{\text{enc}}(R) = \int_0^R \Phi(r) 4\pi r^2 \, dr + e
\]

Let us evaluate the integral:

\[
\int_0^R \Phi(r) 4\pi r^2 \, dr = -\frac{e}{\lambda^2 a^3} \int_0^{R/a} \frac{e^{-2\frac{r}{a}}}{a^2} \, r^2 \, dr
\]

\[
= -\frac{e}{\lambda^2 a^3} \left[ \frac{-25}{a^2} \right] \int_0^{R/a} e^{-2\frac{r}{a}} a^2 \, dr + e
\]

\[
= -\frac{e}{\lambda^2 a^3} \left[ \frac{-25}{a^2} \right] \left[ \frac{-e^{-2R/a}}{2} \right] + e
\]

\[
= -\frac{e}{\lambda^2 a^3} \left[ \left( \frac{R}{a} \right)^2 - 2 \frac{R}{a} \right] + e
\]

\[
= -4e \left[ -\frac{R^2}{2a^2} \frac{e}{2} - \frac{R}{2a} \frac{-R}{2} \frac{e}{2} + \frac{1}{2} \frac{e}{2} \right] + e
\]

\[
= -4e \left[ -2 \frac{R^2}{2a^2} - \frac{R^2}{4a} \frac{e}{2} + \frac{1}{4} \right]
\]

\[
= -4e \left[ e^{-2R/a} \left\{ -\frac{1}{2} \frac{R^2}{2a^2} - \frac{1}{2a} - \frac{1}{2} \right\} + \frac{1}{4} \right]
\]
\[ \int_0^R s(r) 4\pi r^2 \, dr = -e \left[ e^{-2R/a} \left( \frac{2R^2}{a^2} + \frac{2R}{a} + 1 \right) - 1 \right] \]

The total enclosed charge

\[ \Phi_{\text{enc}} = -e^{2R/a} \]

\[ Q_{\text{enc}} = q_e e^{-2} \left[ 1 + \frac{2R}{a} + \frac{2R^2}{a^2} \right] \]

where we changed notation and called \( q_e \) the electronic charge.

The total charge inside a volume of radius \( a \) is

\[ Q_{\text{enc}} (a) = q_e e^{-2} \left[ 1 + 4 \right] \]

\[ Q_{\text{enc}} (a) = q_e 5 e^{-2} \]

(c) By symmetry the electric field will depend only on the radial coordinate and also have only radial component.

\[ \Rightarrow \quad E = E(r) \hat{r} \]
Applying Gauss' law to a surface of a sphere of radius \( r \), we have:

\[
E(r) \ 4\pi r^2 = \frac{1}{\varepsilon_0} \ \Phi_{\text{enc}}(r)
\]

\[
E(r) = \frac{1}{4\pi \varepsilon_0} \ \frac{q_i}{r^2} \left[ 1 + \frac{2r}{a} + \frac{2r^2}{a^2} \right] e^{-2r/a}
\]

as \( r \to 0 \),

\[
E(r) = \frac{q_i}{4\pi \varepsilon_0} \left[ \frac{1}{r^2} + \frac{2}{ar} + \frac{2}{a^2} \right] + \ldots
\]

It is better to plot

\[
\frac{E(r)}{E_{\text{proton}}}
\]
2. (a) \[ E(r) = \frac{e}{4\pi\varepsilon_0 r^2} \left[ 1 + \left( \frac{s}{2e} \right)^2 \right] \]

(b) Zero dipole moment.

(c) \[ V = \frac{1}{4\pi\varepsilon_0} \frac{\Phi}{R} = -0.15 \text{ volt} \]

\[ \Phi = (4\pi\varepsilon_0) R (-0.15) \text{ volt} \]

\[ = (4\pi\varepsilon_0) 3 \times 10^{-7} (-0.15) \text{ C} \]

\[ N = \frac{\Phi}{e} \text{ electronic charge.} \]

3. (b) \[ E(r) = \frac{1}{4\pi\varepsilon_0} \frac{\Phi}{R^2} = \frac{V}{R} \]
(c) \[ F = q (v \times B) \]

\[ \frac{mv^2}{R} = q v B \]

\[ \Rightarrow R = \frac{mv^2}{q B v} = \frac{mv}{q B} \]

(d) \[ T = \frac{2\pi R}{v} = 2\pi \frac{mv}{q B \nu} \]

\[ = 2\pi \frac{m}{q B} \]

3.

(a) Total flux \( \Phi = abB \)

rate of change of flux \( \frac{d\Phi}{dt} = bBv \)

This will induce an emf \( \mathcal{E} = -\frac{d\Phi}{dt} \)
Argument I: Lenz’s law implies that the induced emf will stop the change of flux, so the bar should slow down.

Argument II: A current will be set up. The current passing through resistance R will loose energy. Hence motion should stop.

(b) The current \( I = \frac{E}{R} \) = \( \frac{b BV}{R} \)

The eqn for the metal bar:

\[
m \frac{dv}{dt} = -q v B \]

= \( q b , v B \) 

Charge per unit length.
What is the initial kinetic energy?
\[ \frac{1}{2} m v^2 \]

Initially: \( E = \text{total energy} = \frac{1}{2} m v^2 \)

Rate of change of energy:
\[ \frac{dE}{dt} = \frac{1}{2} m \frac{d}{dt} v^2 \]
\[ = m \frac{v dv}{dt} \]

(rate of energy dissipation) = \[ I^2 R \times \]

(in the resistor)

\[ m \frac{v dv}{dt} = \frac{1}{2} R \]
\[ = \frac{b B^2}{R^2} \]

\[ m \frac{v dv}{dt} = \frac{b B^2}{R^2} \]

\[ \frac{dv}{dt} = \frac{b B^2}{R m} \]
\[ n_f - n_i = \int_0^T \frac{bB}{mR} \, dt \]

\[ 0 - n_0 = \frac{bB^2}{mR} T \]

\[ T = \frac{m \nu R}{bB^2} \]

Moving with constant acceleration:

distance covered

\[ s = \nu t + \frac{1}{2} (\text{acceleration}) t^2 \]

\[ = \nu T - \frac{1}{2} \left( \frac{bB^2}{mR} \right) T^2 \]

(c) Energy is not being conserved but dissipated in the resistor.
(a) \[ \oint \mathbf{dl} \cdot \mathbf{B} = N \mu_0 I \text{enc} \]
\[ B_L = \mu_0 N I \]
\[ B = \mu_0 N I \]

(b) \[ B = 0.4 \text{ Tesla} \]
\[ I = 10 \text{ Amp} \]
\[ N = \frac{B}{\mu_0 I} = \frac{1}{\text{meter}} \]

(c) \[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} \]
\[ 2\pi r E = -\frac{d}{dt} B \pi r^2 \]
\[ = -\pi r^2 \frac{dB}{dt} \]
\[ = -\pi r^2 B_0 \omega \sin \omega t \]
\[ 2\pi r E = \pi r^2 \omega B_0 \]
\[ E = \frac{\pi r \omega B_0}{2\pi} = \frac{\omega r B_0}{2} \]
\[ E = \hat{x} E_0 \sin(y - vt) \]
\[ B = \hat{x} B_0 \sin(y - vt) \]

The Maxwell's equations in free space:

\[ \nabla \cdot E = 0 \]
\[ \nabla \cdot B = 0 \]

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times B = \frac{1}{c^2} \frac{\partial E}{\partial t} \]

Clearly, with the given \( E \) and \( B \), \( \nabla \times E = 0 \)

\[ \nabla \cdot E = 0, \quad \text{and} \quad \nabla \cdot B = 0 \]

\[ \nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & E_0 \sin(y - vt) \end{vmatrix} \]

\[ = \hat{x} E_0 \cos(y - vt) \]

\[ \nabla \times B = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ B_0 \sin(y - vt) & 0 & 0 \end{vmatrix} \]

\[ = -\hat{x} B_0 \sin(y - vt) = -\hat{x} B_0 \cos(y - vt) \]

\[ \frac{\partial E}{\partial t} = +v \hat{z} E_0 \sin \cdot \cos(y - vt) \]

To satisfy \( \nabla \times B = \frac{1}{c^2} \frac{\partial E}{\partial t} \), we have \( B_0 = +v \frac{E_0}{c^2} \).
To satisfy \[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \frac{\partial \mathbf{B}}{\partial t} = -\mathbf{v} \times \mathbf{B}_0 \cos(y-vt) \]

\[\Rightarrow \quad \mathbf{E}_0 = \mathbf{v} \mathbf{B}_0 \]

One solution is \( \mathbf{v} = \mathbf{c}, \quad \mathbf{E}_0 = \mathbf{c} \mathbf{B}_0 \)

(b) A wave propagating in the \(-x\) direction has the equation:

\[ \sin \left( kx + \omega t \right) \]

where

\[ c = \frac{\omega}{k} \text{ is the speed of light.} \]

\[ \omega = 2\pi f = 2\pi \times 100 \times 10^6 \text{ Hz} \]

As the wave is propagating along the \(x\) direction and \(\mathbf{E}\) then \(\mathbf{E}\) must be in the \(y-z\) plane. \(\mathbf{E}\) is perpendicular to \(\hat{z}\) so it must be along \(\hat{y}\)

\[ \mathbf{E} = \hat{y} E_0 \sin \left( kx + \omega t \right) \quad k = \omega c \]

\[ \Rightarrow \quad \mathbf{B} = \hat{z} \mathbf{B}_0 \sin \left( kx + \omega t \right) \]