Solution to Problem Set 1

1.

(a) [diagram showing vector fields]

(b) At the origin

\[ \vec{E} = \left[ -\hat{x} \frac{9}{d^2}, -\hat{y} \frac{9}{d^2} \right] \frac{1}{4\pi \epsilon_0} \]

\[ = \frac{1}{4\pi \epsilon_0} \frac{2q}{d^2} \hat{x} \]

(c) The work necessary is the potential at the point \( P_1 \)

\[ \Phi(P_1) = \frac{1}{4\pi \epsilon_0} \left( \frac{q}{q_1} - \frac{q}{q_2} \right) \]

where \( q_1 = q_2 = \left( \frac{d^2 + h^2}{2} \right)^{\frac{1}{2}} \)

\[ \Rightarrow \Phi(P_1) = 0 \]

The work done is zero.
(d) Clearly, by symmetry

\[ \phi(P_2) = \phi(P_3) = 0 \]

(e) As the potential at \( P_2 \) is equal to that at \( P_3 \), the work necessary is also zero.

Note that all points in the \( y-z \) plane are equidistant from the two charges \( \pm Q \). Hence the potential of all these points are zero. The \( y-z \) plane is an equipotential.

The plane extends to infinity, hence the work done to move any charge from infinity to this plane is also zero.
By symmetry the electric field must be along \( \hat{r} \) the radially outward direction as shown in figure. Also \( \vec{E} \) is a function of \( r \) only. Using the Gaussian surface above, (the contribution from \( A \) and \( A' \) is zero)

\[
2\pi r \hat{r} \cdot \vec{E}(r) = \frac{1}{\varepsilon_0} \int \vec{E} \cdot d\vec{A}
\]

\[
\Rightarrow \quad \vec{E}(r) = \frac{1}{\varepsilon_0} \frac{r^3}{2R^2}
\]

For \( r > 0 \), the total charge enclosed is \( \pi R^2 \), so we obtain

\[
2\pi r \hat{r} \cdot \vec{E}(r) = \frac{1}{\varepsilon_0} \int \vec{E} \cdot d\vec{A}
\]

\[
\Rightarrow \quad \vec{E}(r) = \frac{1}{\varepsilon_0} \left( \frac{3R^2}{2} \right) \frac{1}{r}
\]
3. The potential \( \phi = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R} \) is total charge.

Surface charge density

\[ \sigma = \frac{Q}{4\pi R^2} = \frac{1}{4\pi R^2} \frac{4\pi \varepsilon_0 \phi}{4\pi} \]

\[ \sigma = \frac{\varepsilon_0 \phi}{R} \]

No. of extra electrons

\[ N = \frac{\sigma}{e} = \frac{4\pi \varepsilon_0 \phi}{4\pi R} \frac{1}{e} \]

Electronic charge

\[ = \frac{1}{9 \times 10^9} \frac{10^3 \times 9^2}{4\pi^2 \times 7.5 \times 10^{-10}} \frac{1}{1.6 \times 10^{-19}} \frac{C}{m^2} \]

For a basketball \( 2\pi R = 29.5 \) inches

\[ \approx 30 \text{ inches.} \]

\[ = 30 	imes 2.54 \text{ cm} \]

\[ = \frac{7.5 \times 10}{2\pi} \text{ cm} \]

\[ R = \frac{7.5 \times 10}{2\pi} \text{ cm} \]

\[ = \frac{10^3}{10^{-9}} \frac{1}{9 \times 2 \times 7.5} \text{ C m}^{-2} \]

\[ = 10^{12} \]